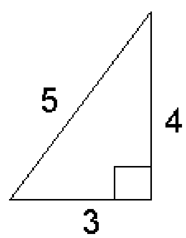
Name:___________________________________ Date:_________________________

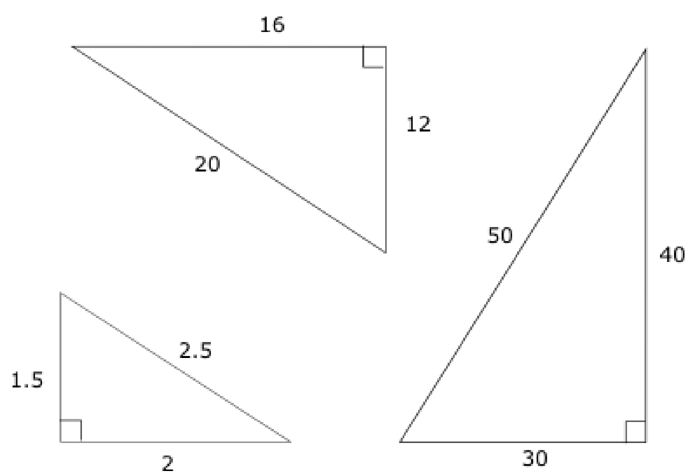
Math 3 Plane Geometry
Review Special Triangles

Special right triangles.

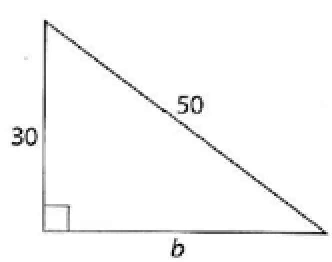
When using the Pythagorean theorem, we often get answers with square roots or long decimals. There are a few special right triangles that give integer answers. The most often seen is the 3-4-5 right triangle. If one leg is 3 and the other is 4 then the hypotenuse will be 5. When you are able to recognize this kind of triangle, you don't even have to use the Pythagorean theorem, which makes things simpler and easier.



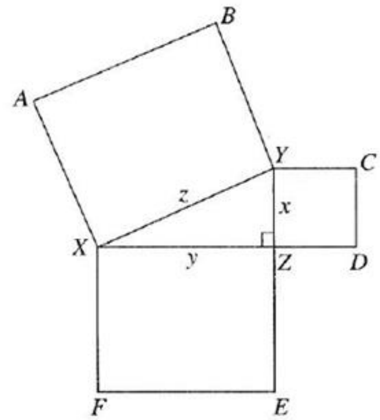
Also, try to recognize multiples of the 3-4-5 right triangle. For example, all the triangles below are variations of the 3-4-5 right triangle. Four times a 3-4-5 right triangle makes a 12-16-20 triangle. Ten times a 3-4-5 makes a 30-40-50 right triangle. One half times a 3-4-5 right triangle makes a 1.5-2-2.5 triangle.



1. In the right triangle below, one leg is 30 and the hypotenuse is 50. Find b.



2. As shown in the figure below, triangle XYZ is a right triangle. Quadrilaterals ABYX, CDZY, and EFXZ are squares. If the measure of side x = 6 inches, and the measure of hypotenuse z = 10 inches, what is the area of quadrilateral EFXZ?

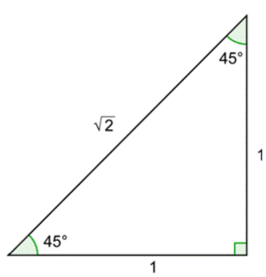


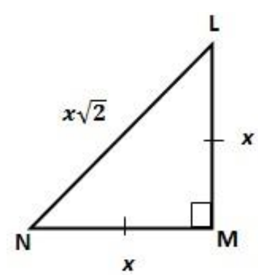
Special right triangles - angles

We've discussed special right triangles that can be recognized by their sides, there are also special right triangles that can be recognized by the angles.

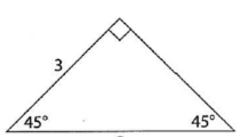
45°-45°-90°

The first is an isosceles right triangle. We know that isosceles triangles have two sides the same length, that means that the angles opposite from those sides are also equal. Therefore every isosceles right triangle has the angles 45°-45°-90°. The sides of a 45°-45°-90° triangle are in a ratio of 1:1: $\sqrt{2}$.

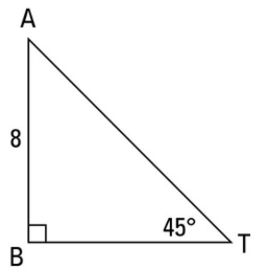




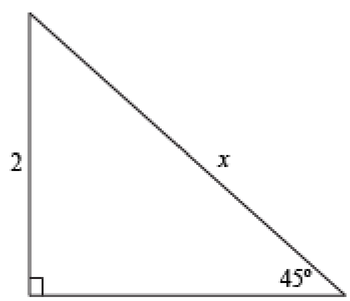
In the example below, we have a 45°-45°-90° triangle. The length of one side is 3. From that we can calculate all the other sides. Since one leg is 3, then the other leg is also 3. To find the hypotenuse multiply the length of the side by $\sqrt{2}$. So in this case it is $3\sqrt{2}$ or about 4.24264...



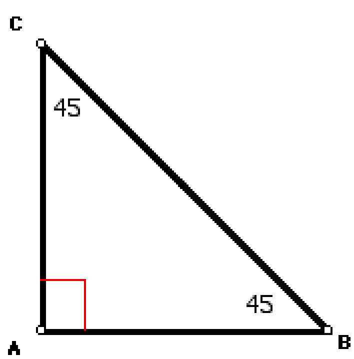
3. Find the length of AT in the triangle below.



4. Solve for x in the triangle below.



5. If the length of side AC is 6, what is the length of side BC?



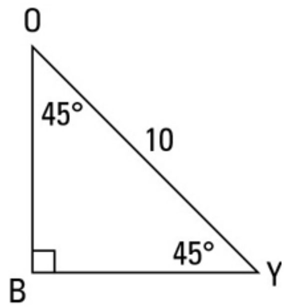
If you know the length of the hypotenuse instead of one of the sides, it's not quite at simple, but still doable. Rather than multiply by $\sqrt{2}$ we will divide by $\sqrt{2}$.

$\frac{x}{\sqrt{2}}$

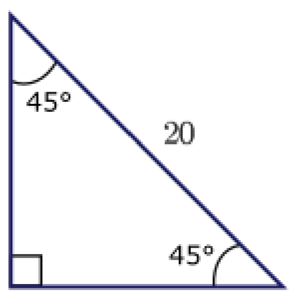
However, we usually don't leave a radical sign as a denominator. In order to get rid of the radical sign in the denominator we multiply both the numerator and the denominator by $\sqrt{2}$. Like this:

$\frac{x}{\sqrt{2}}$ x $\frac{\sqrt{2}}{\sqrt{2}}$ = $\frac{x\sqrt{2}}{2}$

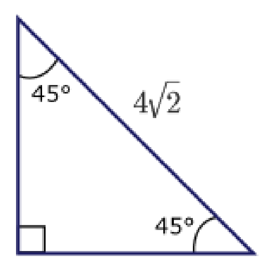
So in other words, if you know the length of the hypotenuse of a 45°-45°-90° triangle, you can find the length of the side by multiplying by $\frac{\sqrt{2}}{2}$. For example. In the 45°-45°-90° triangle below the hypotenuse is 10. To find the length of one of the sides, multiply 10 X $\frac{\sqrt{2}}{2} = 5\sqrt{2}$. Each side is $5\sqrt{2}$ or about 7.07106...



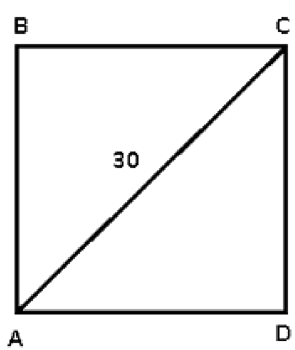
6. In the 45°-45°-90° triangle below the hypotenuse is 20. What is the length of one of the sides?



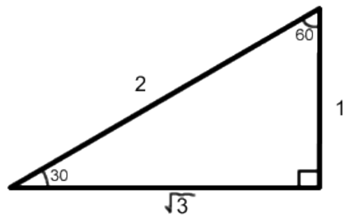
7. In the isosceles right triangle to the right, what is the length of the base leg?

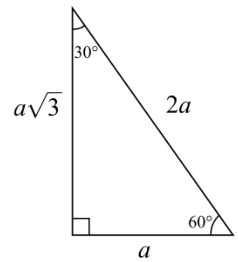


8. A diagonal of a square creates two 45-45-90 triangles. What is the perimeter of square ABCD below?

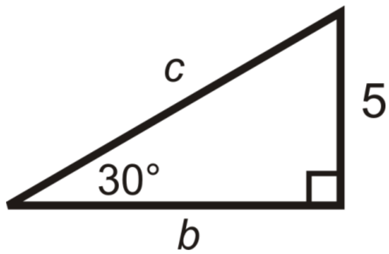


There is another special right triangle with angles 30°-60°-90°. The sides of this triangle are in a ratio of $1 : \sqrt{3} : 2$

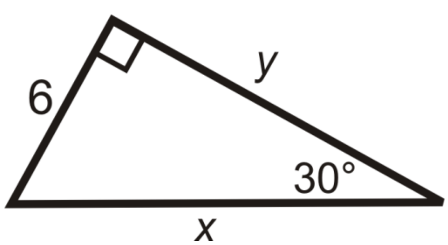




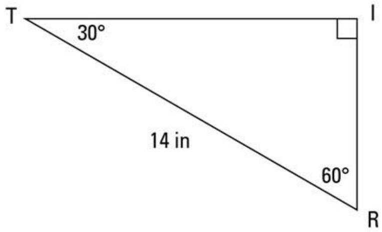
In the triangle below. The length of the short leg is 5. To find the length of the longer leg multiply by $\sqrt{3}$. So it is $5\sqrt{3}$ (or about 8.66025...). To find the length of the hypotenuse, simply multiply the short leg by 2. So the length of the hypotenuse is 10.



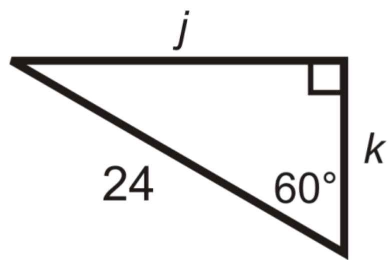
9. Find x and y in the triangle below.



In the triangle below, the length of the hypotenuse is given. From that we can easily calculate the length of the short leg by dividing by 2. So side IR = 7in. To find the length of the longer leg, multiply the length of the shorter leg by $\sqrt{3}$. So side IT = $7\sqrt{3}$ (or about 12.124355...).



10. Find j and k in the triangle below.



If the length of the longer leg is given, it's a little more complicated, but not bad. In the figure below, the length of the longer side is 5. We know that the longer side is the shorter side multiplied by $\sqrt{3}$, so to find the shorter side we divide the longer side by $\sqrt{3}$.

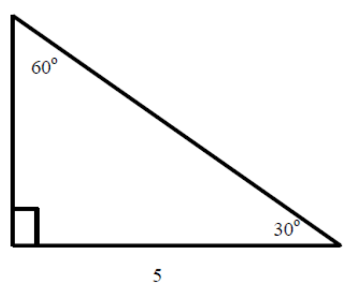
Step 1: $5 = x\sqrt{3}$

Step 2: $\frac{5}{\sqrt{3}} = x$ (divide both sides by $\sqrt{3}$)

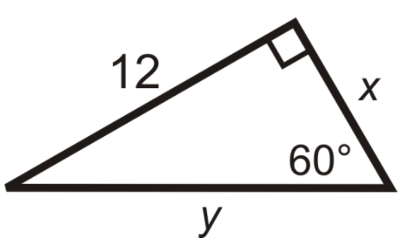
Step 3: $\frac{5\sqrt{3}}{\sqrt{3}\sqrt{3}} = x$ (we never leave a square root sign in the denominator, so, multiply top and bottom by $\sqrt{3}$.

Step 4: $\frac{5\sqrt{3}}{3} = x$ (multiply it out and simplify if possible)

So the length of the shorter side is $\frac{5\sqrt{3}}{3}$ (or about 2.88675...), from there it is easy to find the length of the hypotenuse, simply multiply the shorter side by 2. $\frac{10\sqrt{3}}{3}$



11. Find x and y in the triangle below.



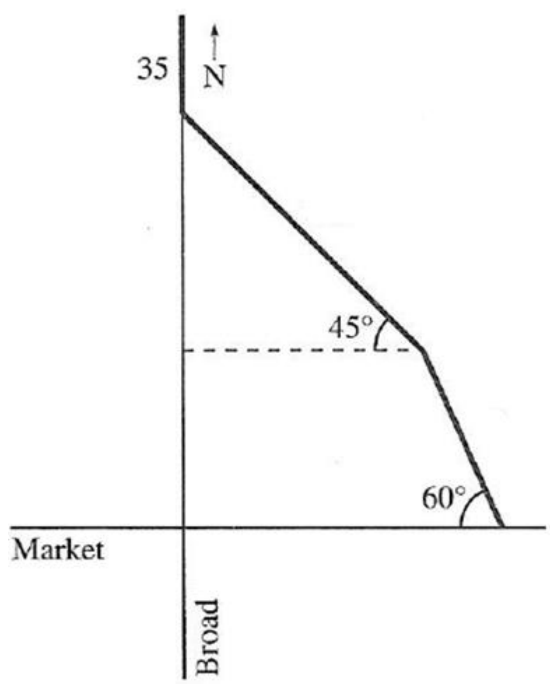
12. What is the perimeter of a 30°-60°-90° triangle with a short leg of 6 inches?

13. What is the perimeter of a 30°-60°-90° triangle with a hypotenuse leg of 24 inches?

14. What is the perimeter of a 30°-60°-90° triangle with a long leg of 5 inches?

15. Cindy wanted to stay off the main road for most of her morning jog. She started on Market Street ran 400 meters along a side road that runs at a 60° angle relative to Market Street. Then the road makes a slight turn so it's at an angle of 45 relative to Market Street and she jogged another 600 meters. At that point the side street joins up with Broad Street and she continued another 35 meters. How far north did she travel during her morning jog?

A. 35
B. 115
C. 195
D. $50\sqrt{2} + 30\sqrt{3}$
E. $35 + 300\sqrt{2} + 200\sqrt{3}$



Answers

1. 40
2. 64 in^2
3. $8\sqrt{2}$
4. $2\sqrt{2}$
5. $6\sqrt{2}$
6. $10\sqrt{2}$
7. 4
8. $60\sqrt{2}$ or 84.85
9. x = 12; y = $6\sqrt{3}$
10. j = $12\sqrt{3}$; k = 12
11. x = $4\sqrt{3}$; y = $8\sqrt{3}$
12. 18 + $6\sqrt{3}$ or 28.39 inches
13. 36 + $12\sqrt{3}$ or 56.78 inches
14. 5 + $5\sqrt{3}$ or 13.66 inches
15. E