

## Math 3 Proportion & Probability Part 2

### Sequences, Patterns, Frequency Tables & Venn Diagrams

#### MATH 2 REVIEW

##### ARITHMETIC SEQUENCES

In an Arithmetic Sequence the difference between one term and the next is a constant. In other words, we just **add** the same value each time ... infinitely. In general, we could write an arithmetic sequence like this:

$$\{a, a+d, a+2d, a+3d, \dots\}$$

where:  $a$  is the first term, and  $d$  is the difference between the terms (called the "common difference")

In the example:  $a = 1$  (the first term) and  $d = 3$  (the "common difference" between terms)

And we get:

$$\{a, a+d, a+2d, a+3d, \dots\}$$

$$\{1, 1+3, 1+2 \times 3, 1+3 \times 3, \dots\}$$

$$\{1, 4, 7, 10, \dots\}$$

##### EXPLICIT FORMULA FOR ARITHMETIC SEQUENCES

We can write an Arithmetic Sequence as an explicit formula:

$$a_n = a_1 + d(n-1) \quad (\text{We use "n-1" because d is not used in the 1st term}).$$

**Example:** Write the explicit formula for arithmetic sequence, and calculate the 4th term for 3, 8, 13, 18, 23, 28, 33, 38, . . . .

**Solution:** This sequence has a difference of 5 between each number.

The values of  $a$  and  $d$  are:  $a = 3$  (the first term) and  $d = 5$  (the "common difference")

The explicit formula can be calculated:

$$a_n = a_1 + d(n-1) = 3 + 5(n-1) = 3 + 5n - 5 = 5n - 2$$

$$\text{So, the 4th term is: } a_4 = 5(4) - 2 = 18$$

##### RECURSIVE FORMULA FOR ARITHMETIC SEQUENCES

We can also write an Arithmetic Sequence as a recursive formula:

$$a_n = a_{n-1} + d \quad (\text{We use "n-1" because d is not used in the 1st term}).$$

**Example:** Write the recursive formula for arithmetic sequence, and calculate the 4th term for 3, 8, 13, 18, 23, 28, 33, 38, . . . .

**Solution:** As from above, this sequence has a difference of 5 between each number.

The values of  $a$  and  $d$  are:  $a = 3$  (the first term) and  $d = 5$  (the "common difference")

The recursive formula can be calculated:

$$a_n = a_{n-1} + d \quad a_2 = a_1 + 5 = 3 + 5 = 8 \quad a_3 = a_2 + 5 = 8 + 5 = 13$$

$$\text{So, the 4th term is: } a_4 = a_3 + 5 = 13 + 5 = 18$$

Sample Question:

1. Which of the following statements is NOT true about the arithmetic sequence 16, 11, 6, 1. . . ?
  - a. The fifth term is -4.
  - b. The sum of the first 5 terms is 30.
  - c. The seventh term is -12
  - d. The common difference of consecutive integers is -5.
  - e. The sum of the first 7 terms is 7.
2. The greatest integer of a set of consecutive even integers is 12. If the sum of these integers is 40, how many integers are in this set?
3. What is the sum of the first 4 terms of the arithmetic sequence in which the 6th term is 8 and the 10th term is 13?

## GEOMETRIC SEQUENCE

In a Geometric Sequence each term is found by **multiplying** the previous term by a constant.

For illustration, look at the sequence: 2, 4, 8, 16, 32, 64, 128, 256, . . .

This sequence has a factor of 2 between each number. Each term (except the first term) is found by multiplying the previous term by 2. In general, we write a Geometric Sequence like this:  $\{a, ar, ar^2, ar^3, \dots\}$  where:  $a$  is the first term, and  $r$  is the factor between the terms (called the "common ratio")

**Example:**  $\{1, 2, 4, 8, \dots\}$

**Solution:** The sequence starts at 1 and doubles each time, so  $a=1$  (the first term) and  $r=2$  (the "common ratio" between terms is a doubling)

And we get:

$$\{a, ar, ar^2, ar^3, \dots\} = \{1, 1 \times 2, 1 \times 2^2, 1 \times 2^3, \dots\} = \{1, 2, 4, 8, \dots\}$$

But be careful,  $r$  should not be 0: When  $r = 0$ , we get the sequence  $\{a, 0, 0, \dots\}$  which is not geometric

## EXPLICIT FORMULA FOR GEOMETRIC SEQUENCES

We can also calculate any term using the explicit formula:

$$a_n = a_1 r^{(n-1)} \quad (\text{We use "n-1" because } ar^0 \text{ is for the 1st term})$$

**Example:** 10, 30, 90, 270, 810, 2430, . . . .

**Solution:** This sequence has a factor of 3 between each number.

The values of  $a$  and  $r$  are:  $a = 10$  (the first term) and  $r = 3$  (the "common ratio")

The Rule for any term is:

$$a_n = 10 \times 3^{(n-1)} \quad \text{So, the 4th term is: } a_4 = 10 \times 3^{(4-1)} = 10 \times 3^3 = 10 \times 27 = 270$$

$$\text{And the 10th term is: } a_{10} = 10 \times 3^{(10-1)} = 10 \times 3^9 = 10 \times 19683 = 196830$$

## RECURSIVE FORMULA FOR GEOMETRIC SEQUENCES

We can also calculate a geometric sequence using the recursive formula:

$$a_n = a_{n-1}(r)$$

**Example:** 10, 30, 90, 270, 810, 2430, . . . .

**Solution:** As listed above, this sequence has a factor of 3 between each number.

The values of  $a$  and  $r$  are:  $a = 10$  (the first term) and  $r = 3$  (the "common ratio")

$$a_2 = a_1(r) = 10(3) = 30 \quad a_3 = a_2(r) = 30(3) = 90 \quad a_4 = a_3(r) = 90(3) = 270$$

**Example:** 4, 2, 1, 0.5, 0.25, . . . .

**Solution:** This sequence has a factor of 0.5 (a half) between each number.

Using the explicit formula:  $a_n = 4 \times (0.5)^{n-1}$  Using the recursive formula:  $a_n = a_{n-1}(0.5)$

Example Questions:

4. The first term is 1 in the geometric sequence 1, -3, 9, -27, . . . . What is the SEVENTH term in the geometric sequence?
5. The first and second terms of a geometric sequence are  $a$  and  $ab$ , in that order. What is the 643<sup>rd</sup> term of the sequence?
6. Which of the following statements is NOT true about the geometric sequence 26, 18, 9, . . . ?
  - a. The fourth term is 4.5
  - b. The sum of the first five terms is 59.75
  - c. Each consecutive term is  $\frac{1}{2}$  of the previous term
  - d. Each consecutive term is evenly divisible by 3
  - e. The common ratio of consecutive terms is 2:1

### COUNTING CONSECUTIVE INTEGERS

To find the number of consecutive integers between two values, **subtract the smallest from the largest and add 1**. So to find the number of consecutive integers from 13 through 31, subtract:  $31 - 13 = 18$ . Then add 1:  $18 + 1 = 19$ . There are 19 consecutive integers from 13 through 31.

### USING THE SLOT METHOD TO FIND CONSECUTIVE INTEGERS

First write the sequence including the unknown numbers by leaving slots to fill in the numbers. Then find the “common difference” between the first and last terms in the sequence. Divide the difference by the number of “jumps” between the first and last integer.

**Example:** What two numbers should be placed in the blanks below so that the difference between successive entries is the same?

26, \_\_\_\_\_, \_\_\_\_\_, 53

- 36, 43
- 35, 44
- 34, 45
- 33, 46
- 30, 49

**Solution:** Common Difference =  $53 - 26 = 27$ . There are three jumps (26 to 2<sup>nd</sup> term, 2<sup>nd</sup> term to 3<sup>rd</sup> term, 3<sup>rd</sup> term to 53) so divide 27 by 3 = 9. The second term would be  $26 + 9 = 35$ . The third term would be  $35 + 9 = 44$  (B).

### USING ARITHMETIC SEQUENCE TO FIND CONSECUTIVE INTEGERS

**Example:** What two numbers should be placed in the blanks below so that the difference between successive entries is the same?

26, \_\_\_\_\_, \_\_\_\_\_, 53

**Solution:** In the arithmetic sequence, you can think of the terms as 26,  $26 + s$ ,  $26 + s + s$ , and  $26 + s + s + s$ . In the example,  $s$  represents the difference between successive terms. The final term is 53, so set up an algebraic equation:  $26 + s + s + s = 53$ . Solve the equation for  $s$ , by first combining like terms:  $26 + 3s = 53$ . Subtract 26 from both sides to get  $3s = 27$ . Divide both sides by 3 to find that  $s$ , the difference between terms is 9. Therefore, the terms are 26,  $26 + 9$ ,  $26 + 9 + 9$ , and 53 or 26, 35, 44, and 53 (B)

Sample Questions:

7. For the difference between consecutive numbers to be the same, which 3 numbers must be placed in the blanks below?

11, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, 47

8. What two numbers should be placed in the blanks below so that the difference between the consecutive numbers is the same?

13, \_\_\_\_\_, \_\_\_\_\_, 34

## SUMMING AN ARITHMETIC SERIES USING A FORMULA

To sum up the terms of this arithmetic sequence:

$$a + (a+d) + (a+2d) + (a+3d) + \dots$$

$$\sum_{k=0}^{n-1} (a + kd) = \frac{n}{2}(2a + (n-1)d)$$

a is the first term

d is the "common difference" between terms

n is the number of terms to add up

$\sum$  (called Sigma) means "sum up"

And below and above it are shown the starting and ending values:

$$\sum_{n=1}^4 n = 1+2+3+4 = 10$$

It says "Sum up n where n goes from 1 to 4. Answer=10"

**Example:** Add up the first 10 terms of the arithmetic sequence:

$$\{ 1, 4, 7, 10, 13, \dots \}$$

**Solution:** The values of a, d and n are:

a = 1 (the first term)

d = 3 (the "common difference" between terms)

n = 10 (how many terms to add up)

$$\text{So: } \sum_{k=0}^{n-1} (a + kd) = \frac{n}{2}(2a + (n-1)d)$$

$$\text{Becomes: } \sum_{k=0}^{10-1} (1 + k \cdot 3) = \frac{10}{2}(2 \cdot 1 + (10-1) \cdot 3) = 5(2+9 \cdot 3) = 5(29) = 145$$

**Check:** Add up the terms yourself using a calculator, and see if it comes to 145

**Example:** Cynthia decorates the ceiling of her bedroom with stars that glow in the dark. She puts 1 star on the ceiling on the first day of decorating, 2 stars on the ceiling on the 2<sup>nd</sup> day of decorating, 3 stars on the 3<sup>rd</sup> day, and so on. If she puts stars on the ceiling in this pattern for 30 days (so she puts 30 stars on the ceiling on the 30<sup>th</sup> day), then what will be the total number of stars on the ceiling at the end of 30 days?

**Solution:** You can use your calculator to add  $1 + 2 + 3 + 4 + \dots + 30 = 465$ . Or if you identified this as an arithmetic series, you can use the equation:

$$\sum_{k=0}^{n-1} (a + kd) = \frac{n}{2}(2a + (n-1)d)$$

Where a = 1, d = 1, n = 30

Becomes:

$$\sum_{k=0}^{n-1} (1 + k1) = \frac{30}{2}[2(1) + (30-1)(1)] = 15 \times 31 = 465$$

The **simplified equation** for finding the sum of arithmetic sequences is:

$$S = (a_1 + a_n) \times \frac{n}{2}$$

a<sub>1</sub> is the first term

a<sub>n</sub> is the last term

n is the number of terms to add up

**Example:** Cynthia decorates the ceiling of her bedroom with stars that glow in the dark. She puts 1 star on the ceiling on the first day of decorating, 2 stars on the ceiling on the 2<sup>nd</sup> day of decorating, 3 stars on the 3<sup>rd</sup> day, and so on. If she puts stars on the ceiling in this pattern for 30 days (so she puts 30 stars on the ceiling on the 30<sup>th</sup> day), then what will be the total number of stars on the ceiling at the end of 30 days?

**Solution:** This time, use the simplified formula  $S = (a_1 + a_n) \times \frac{n}{2}$ , where  $n$  is the number of terms (30),  $a_1$  is the first term (1),  $a_n$  is the  $n$ th term (30).

$$(1 + 30) \times \frac{30}{2} = 465.$$

Sample Questions:

9. A theater has 50 rows of seats. There are 18 seats in the first row, 20 seats in the second, 22 in the third and so on. How many seats are in the theater?
10. The eleventh term of an arithmetic sequence is 30 and the sum of the first eleven terms is 55. What is the common difference?
11. On the first day of school, Mr. Vilani gave his third-grade students 5 new words to spell. On each day of school after that, he gave the students 3 new words to spell. In the first 20 days of school, how many new words had he given the students to spell?
12. How many terms of the arithmetic sequence 2, 8, 14, 20, ..are required to give a sum of 660?

### SUMMING AN ARITHMETIC SERIES WITH SLOT FORMULA OR CONSECUTIVE INTEGERS

To solve via slot method, draw appropriate number of slots and figure out how the numbers in the slots are related. Figure out a formula or other solution to the problem and solve. Consecutive integers can be written as  $n, n+1, n+2$ , etc. Consecutive odd or even integers can be written as  $n, n+2, n+4$ , etc.

**Example:** The average of 7 consecutive numbers is 16. What is the sum of the least and greatest of the 7 integers?

Solution:  $\underline{n + (n + 1) + (n + 2) + (n + 3) + (n + 4) + (n + 5) + (n + 6)} = 16$  or  $\frac{7n+21}{7} = 16$

Multiply both sides by 7 and subtract 21 from each side.  $7n = 91$  Divide both sides by 7 and get  $n = 13$ . So, the first term is 13. The series is 13, 14, 15, 16, 17, 18, 19. The sum of  $13 + 19 = 32$

Sample Questions:

13. The 6 consecutive integers below add up to 513.  
 $n - 2$   
 $n - 1$   
 $n$   
 $n + 1$   
 $n + 2$   
 $n + 3$   
 What is the value of  $n$ ?
14. What is the smallest possible value for the product of 2 real numbers that differ by 6?
15. The product of two consecutive integers is between 137 and 159. Which of the following CAN be one of the integers?  
 a. 15  
 b. 13  
 c. 11  
 d. 10  
 e. 7

## SUMMING A GEOMETRIC SERIES

When we need to sum a Geometric Sequence, there is a handy formula.

To sum:  $a + ar + ar^2 + \dots + ar^{n-1}$  Each term is  $ar^k$ , where  $k$  starts at 0 and goes up to  $n-1$

Use this formula:

$$\sum_{k=0}^{n-1} (ar^k) = a \left( \frac{1 - r^n}{1 - r} \right)$$

$a$  is the first term

$r$  is the "common ratio" between terms

$n$  is the number of terms

**Example:** Sum the first 4 terms of 10, 30, 90, 270, 810, 2430, ...

This sequence has a factor of 3 between each number. The values of  $a$ ,  $r$  and  $n$  are:

$a = 10$  (the first term)     $r = 3$  (the "common ratio")     $n = 4$  (we want to sum the first 4 terms)

So:  $\sum_{k=0}^{n-1} (ar^k) = a \left( \frac{1 - r^n}{1 - r} \right)$       Becomes:  $\sum_{k=0}^{4-1} (10 \cdot 3^k) = 10 \left( \frac{1 - 3^4}{1 - 3} \right) = 400$

And, yes, it is easier to just add them in this example, as there are only 4 terms. But imagine adding 50 terms ... then the formula is much easier.

The **simplified equation** for finding the sum of geometric sequences looks very similar. If  $S_n$  represents the sum of the first  $n$  terms of a geometric series, then

$$S_n = a + ar + ar^2 + ar^3 + ar^4 + \dots + ar^{n-1}$$

Where  $a$  = the first term,  $r$  = the common ratio, and the  $n$ th term is not  $ar^n$ , but  $ar^{n-1}$

The sum of the first  $n$  terms is given by  $S_n$  where  $S_n = \frac{a(1-r^n)}{1-r}$

The sum to infinity, written as  $S_\infty$  is given by  $S_\infty = \frac{a}{1-r}$  only if  $-1 < r < 1$

**Example:** The sum of an infinite geometric sequence series with first term  $x$  and common ratio  $y < 1$  is given by  $\frac{x}{(1-y)}$ . The sum of a given infinite geometric series is 200, and the common ratio is 0.15. What is the second term of this series?

**Solution:**  $S_\infty = \frac{a}{1-r}$        $200 = \frac{x}{1-0.15} = 170$        $170 * 0.15 = 25.5$

Sample Questions:

16. Find the sum of each of the geometric series  $-2, \frac{1}{2}, -\frac{1}{8}, \dots, -\frac{1}{37268}$

17. Work out the sum  $\sum_{r=1}^{\infty} \left(\frac{1}{3}\right)^r$

18. Find the sum of the first ten terms of the geometric sequence:  $a_n = 132(-2)^{n-1}$ .

19. If the sum of the first seven terms in a geometric series is 2158 and  $r = -12$ , find the first term.

## TYPES OF DATA AND TABLES

Variables that take on values that are names or labels are considered categorical data. These are data that cannot be averaged or represented by a scatter plot as they have no numerical meaning. Variables that represent a measurable quantity are numerical or quantitative data.

**Example:** Answer the question. If it can't be answered, explain why. The temperature at the park over a 12-hour period was: 60, 64, 66, 71, 75, 77, 78, 80, 78, 77, 73, and 65. Find the average temperature over the 12-hour period.

**Solution:** The data is quantitative so it is possible to find the average of 72

**Example:** Once a week, Maria has to fill her car up with gas. She records the day of the week each time she fills her car for three months. Monday, Friday, Tuesday, Thursday, Monday, Wednesday, Monday, Tuesday, Wednesday, Tuesday, and Thursday. Find the average day of the week that Maria had to fill her car with gas.

**Solution:** The data is categorical so the average cannot be calculated. However, it is clear that Monday was the day that she most often filled her car.

Sample Questions:

20. A group of students has ages of 12, 15, 11, 14, 12, 11, 15, 13, 12, 12, 11, 14, and 13. Find the average age of the students.

21. Suppose 11-year-olds are in 6th grade, 12-year-olds are in 7th grade, 13-year-olds are in 8th grade, 14-year-olds are in 9th grade, and 15-year-olds are in 10th grade. A group of students has ages of 12, 15, 11, 14, 12, 11, 15, 13, 12, 12, 11, 14, and 13. Find the average grade of the students.

## TWO-WAY FREQUENCY TABLE

A two-way frequency table is a useful tool for examining the relationships between categorical variables. The entries in the cells of a two-way table can be frequency counts or relative frequencies. The table summarizes and shows how often a value occurs.

Generally, conditional relative frequencies are written as a decimal or percentage. It is the ratio of the observed number of a particular event to the total number of events, often taken as an estimate of probability. You can use the relative frequency to determine how often a value may occur in the future.

In a two-way table, entries in the "total" row and "total" columns are called marginal frequencies. Entries in the body of the table are called joint frequencies.

The two-way frequency table below shows the favorite leisure activities for 20 men and 30 women using frequency counts.

	Dance	Sports	Movies	Total
Women	16	6	8	30
Men	2	10	8	20
Total	18	16	16	50

The two-way frequency table below shows the same information represented as conditional relative frequencies.

	Dance	Sports	Movies	Total
Women	0.32	0.12	0.16	0.60
Men	0.04	0.20	0.16	0.40
Total	0.36	0.32	0.32	1.00

Two-way tables can show relative frequencies for the whole table, for rows, or for columns.

Relative Frequency of Rows					Relative Frequency of Columns				
	Dance	Sports	Movies	Total		Dance	Sports	Movies	Total
Women	0.53	0.20	0.27	1.00	Women	0.89	0.38	0.50	0.60
Men	0.10	0.50	0.40	1.00	Men	0.11	0.62	0.50	0.40
Total	0.36	0.32	0.32	1.00	Total	1.00	1.00	1.00	1.00

Each type of relative frequency table makes a different contribution to understanding the relationship between gender and preferences for leisure activity. For example, the “Relative Frequency of Rows” table most clearly shows the probability that each gender will prefer a particular leisure activity. For instance, it is easy to see that the probability that a man will prefer movies is 40% while it is 27% for women.

**Example:** A public opinion survey explored the relationship between age and support for increasing the minimum wage. The results are found in the following two-way frequency table.

	For	Against	No Opinion	TOTAL
Ages 21 – 40	25	20	5	50
Ages 41 – 60	30	30	15	75
Over 60	50	20	5	75
TOTAL	105	70	25	200

In the 41 to 60 age group, what percentage supports increasing the minimum wage? Answer: A total of 75 people in the 41 to 60 age group were surveyed. Of those, 30 were for increasing the minimum wage. Thus, 40% supported increasing the minimum wage.

**Example:** Create two-way frequency tables to organize the information from the situations described.

A bank teller splits transactions into two categories: deposits and withdrawals. In one shift, the bank teller has 72 transactions. Of those, 12 males make deposits and 30 make withdrawals. While 20 females make deposits.

**Solution:** First, draw an empty frequency table and add the appropriate labels.

	Deposits	Withdrawals	Total
Men			
Women			
Total			

Then add the numbers that are listed in the question to the table.

	Deposits	Withdrawals	Total
Men	12	30	
Women	20		
Total			72

Finally, calculate the missing numbers and add them to the table. Verify that all columns and rows add up correctly. Total Deposits:  $12 + 20 = 32$ . If the total transactions is 72, there must be  $72 - 32 = 40$  withdrawals. To get a total of 40 withdrawals, there must be  $40 - 30 = 10$  by women. That makes a total of  $20 + 10 = 30$  transactions by women. Total transactions by men =  $12 + 30 = 42$ . If you add the total transactions by men and women, you get  $42 + 30 = 72$ . Which is verified by adding the total deposits and withdrawals  $32 + 40 = 72$

	Deposits	Withdrawals	Total
Men	12	30	42
Women	20	10	30
Total	32	40	72

Sample Questions:

22. Complete the two-way frequency table and answer the questions about the information in the table  
Hollywood Junior students were interviewed about their eating habits.

	Male	Female	Total
Eat Breakfast Regularly		110	300
Don't Eat Breakfast Regularly	130	165	
Total		275	

- Of the total males, what percentage do not eat breakfast regularly?
- Of the total people who eat breakfast regularly, what percentage of them are males?
- How many females eat breakfast regularly?
- How many females were included in the survey?
- How many females eat breakfast out of the total number of females?
- How many people were included in this survey?
- How many males do not eat breakfast regularly?
- How many males and females do not eat breakfast regularly?
- Which group of people eat breakfast more regularly?

23. Complete the two-way frequency table and answer the questions about the information in the table.  
Jersey High students were surveyed about the type of transportation they use the most often.

	Male	Female	Total
Walk		46	
Car	28		45
Bus		12	27
Cycle		17	69
Total	129	92	

- Of the total males, what percentage cycle?
- Of the total people who use the bus, what percentage are female?
- How many more females than males walk?
- What is the percent of males that walk compared to ride something?
- How many females ride in a car?
- How many total people surveyed?
- How many total males cycle?

**VENN DIAGRAMS**

An event is an activity or experiment which is usually represented by a capital letter. A sample space is a set of all possible outcomes for an activity or experiment. A smaller set of outcomes from the sample space is called a subset. The complement of a subset is all outcomes in the sample space that are not part of the subset. A subset and its complement make up the entire sample space. If a subset is represented by A, the complement can be represented by any of the following: not A,  $\sim A$ , or  $A^c$ .

Event	Sample Space	Possible Subset	Complement
Flip a coin	$S = \{\text{heads, tails}\}$	$B = \{\text{heads}\}$	$B^c = \{\text{tails}\}$
Roll a die	$S = \{1, 2, 3, 4, 5, 6\}$	even $E = \{2, 4, 6\}$	$\sim E = \{1, 3, 5\}$
Pick a digit 0-9	$S = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$	$N = \{2, 5, 7, 9\}$	not N = $\{0, 1, 3, 4, 6, 8\}$

Definition	Example	Venn Diagram
The <b>union</b> of two events includes all outcomes from each event. The union can be indicated by the word "or" or the symbol $\cup$ .	$A = \{0, 2, 4, 6, 8, 10\}$ $B = \{0, 5, 10, 15, 20\}$ $A \cup B = \{0, 2, 4, 5, 6, 8, 10, 15, 20\}$	
The <b>intersection</b> of two events includes only those outcomes that are in both events. The intersection can be indicated by the word "and" or the symbol $\cap$ . If the two events do NOT have anything in common, the intersection is the "empty set", indicated by $\{ \}$ or $\emptyset$ .	$A = \{0, 2, 4, 6, 8, 10\}$ $B = \{0, 5, 10, 15, 20\}$ $A \cap B = \{0, 10\}$	

	<p>The sample space is <math>S = \{\text{Green, Violet, Turquoise, Yellow, Blue, Red, White, Brown, Peach, Black, Magenta, Orange}\}</math></p> <p>The subset of primary colors is <math>P = \{\text{Yellow, Blue, Red}\}</math></p> <p>The subset of American Flag colors is: <math>A = \{\text{Blue, Red, White}\}</math></p> <p><math>P \cup A = \{\text{Yellow, Blue, Red, White}\}</math></p> <p><math>P \cap A = \{\text{Blue, Red}\}</math></p> <p><math>P^c = \{\text{Green, Violet, Turquoise, White, Brown, Peach, Black, Magenta, Orange}\}</math></p> <p><math>\sim(P \cup A) = \{\text{Green, Violet, Turquoise, Brown, Peach, Black, Magenta, Orange}\}</math></p>
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## Sample Questions:

24. Given all the letters of the English Alphabet

- a. Draw a Venn Diagram of the following:
  - i. List a subset of the letters in your first name.
  - ii. List a subset of the letters in your last name.
- b. Find the union of the subsets of your first name and last name.
- c. Find the intersection of the subsets of your first name and last name.

25. Given the sample space  $S = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$  with event  $A = \{3, 4, 5, 6, 7\}$  and event  $B = \{1, 2, 3, 4, 5\}$ .

- a. Draw a Venn diagram representing the sample space with events A and B.
- b. List all the outcomes for  $A \cup B$ .
- c. List all the outcomes for  $A \cap B$ .

26. Given a standard deck of 52 cards, event A is defined as a red card and event B is defined as the card is a diamond.

- a. List all the outcomes for  $A \cup B$ .
- b. List all the outcomes for  $A \cap B$ .
- c. What is  $A \cap A^c$ ? ( $A^c$  is the compliment or opposite data of A)

## Answer Key

- |   |   |
|---|---|
| <ol style="list-style-type: none"> <li>1. C</li> <li>2. 5 numbers</li> <li>3. 14.5</li> <li>4. 729</li> <li>5. <math>ab^{642}</math></li> <li>6. D</li> <li>7. 20, 29, 38</li> <li>8. 20, 27</li> </ol> | <ol style="list-style-type: none"> <li>9. 3,350</li> <li>10. Common Difference = 5</li> <li>11. 62 words</li> <li>12. 15 terms</li> <li>13. 85</li> <li>14. -9</li> <li>15. 13</li> </ol> |
|---|---|

16. .  $a_1 = -2; a_2 = \frac{1}{2}; a_3 = -\frac{1}{8}; a_n = -\frac{1}{37268}$       write down key terms

$$r = \frac{a_2}{a_1} = \frac{\frac{1}{2}}{-2} = -\frac{1}{4}$$

find r using  $r = \frac{a_2}{a_1}$

$$-\frac{1}{37268} = (-2) \left(-\frac{1}{4}\right)^{n-1}$$

substitute the values of  $a_1, a_n$  and r to  $a_n = a_1 r^{n-1}$  to find n

$$\frac{1}{65536} = \left(-\frac{1}{4}\right)^{n-1}$$

divide both sides by -2

$$\left(-\frac{1}{4}\right)^8 = \left(-\frac{1}{4}\right)^{n-1}$$

change  $\frac{1}{65536}$  to its exponential form whose base=r

$$8 = n - 1$$

equate the indices since they both have the same base

$$8 + 1 = n$$

add 1 to both sides

$$9 = n$$

add 8 and 1

$$S_9 = \frac{-2 \left(1 - \left(-\frac{1}{4}\right)^9\right)}{1 - \left(-\frac{1}{4}\right)}$$

substitute the values of  $a_1, r$  and  $n$  to  $S_n = \frac{a_1(1-r^n)}{1-r}$

$$= \frac{1 \left(1 - \left(-\frac{1}{262144}\right)\right)}{\frac{5}{4}}$$

evaluate  $\left(-\frac{1}{4}\right)^9$  then subtract  $-\frac{1}{4}$  from 1

$$= \frac{262145}{\frac{5}{4}}$$

evaluate  $\left(1 - \left(-\frac{1}{262144}\right)\right)$

$$S_9 = \frac{52429}{60536}$$

divide  $\frac{262145}{262144}$  by  $\frac{5}{4}$

17.  $5\infty = \frac{1}{2}$

18.  $\frac{341}{32}$

19. .

20. The data is quantitative so it is possible to find the average age of 12.69.

21. Even though the grade level is a number the data is categorical so the average grade level cannot be found. However, is clear that 7th grade has the highest number of students.

22.

	Male	Female	Total
Eat Breakfast Regularly	190	110	300
Don't Eat Breakfast Regularly	130	165	295
Total	320	275	595

- a. 41%
- b. 63%
- c. 110
- d. 300
- e. 110
- f. 595
- g. 130
- h. 320
- i. male

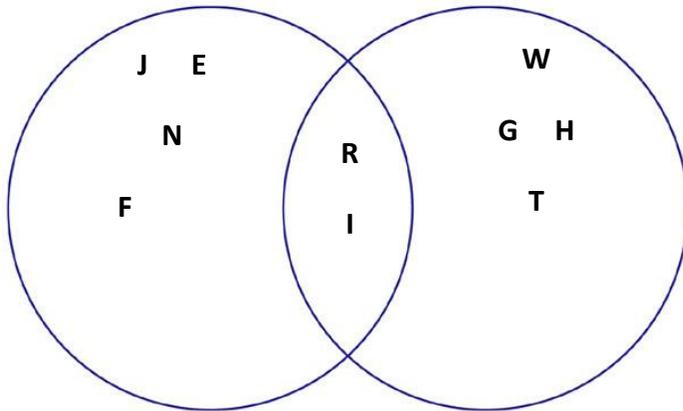
23.

	Male	Female	Total
Walk		46	
Car	28		45
Bus		12	27
Cycle		17	69
Total	129	92	

- a. 40%
- b. 44%
- c. 12
- d. 36%
- e. 17
- f. 221
- g. 52

24. Example: Jennifer Wright

a.



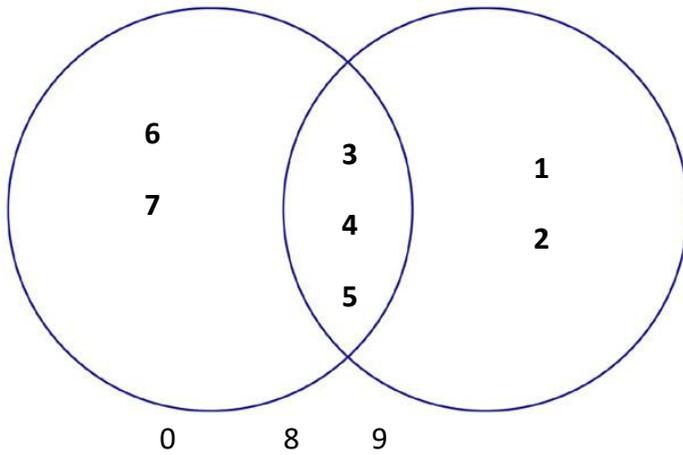
A B C D K L M O P Q S U V X Y Z

b. Union: E, F, G, H, I, J, N, R, T, W

c. Intersection: R, I

25.

d.



e. {1, 2, 3, 4, 5, 6, 7}

f. {3, 4, 5}

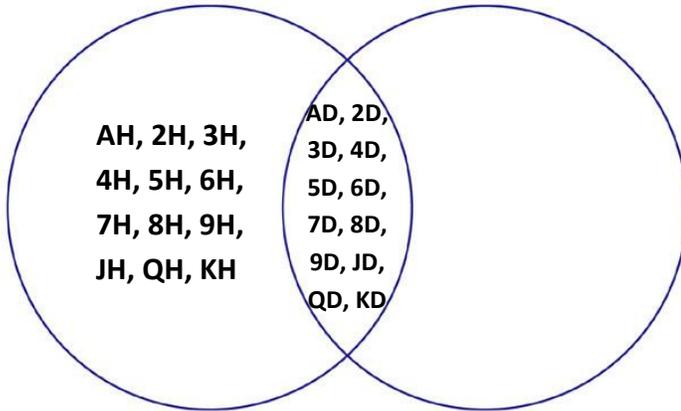
g. {0, 1, 2, 8, 9}

26. A = ace  
K = king

Q = queen  
J = jack

H = heart  
D = diamond

S = spade  
C = club



AS, 2S, 3S, 4S, 5S, 6S, 7S, 8S, 9S, JS, QS, KS, AC, 2C,  
3C, 4D, 5C, 6C, 7C, 8C, 9C, JC, QC, KC

- AH, 2H, 3H, 4H, 5H, 6H, 7H, 8H, 9H, JH, QH, KH, AD, 2D, 3D, 4D, 5D, 6D, 7D, 8D, 9D, JD, QD, KD
- AD, 2D, 3D, 4D, 5D, 6D, 7D, 8D, 9D, JD, QD, KD
- AD, 2D, 3D, 4D, 5D, 6D, 7D, 8D, 9D, JD, QD, KD, AS, 2S, 3S, 4S, 5S, 6S, 7S, 8S, 9S, JS, QS, KS, AC, 2C, 3C, 4D, 5C, 6C, 7C, 8C, 9C, JC, QC, KC
- NONE