# Math 1 Variable Manipulation Part 7
# Linear Functions

**FUNCTION NOTATION**

Traditionally, functions are referred to by the notation *f(x)* read as "*f of x*." However, *f* need not be the only letter used in function names; *g(x), h(x)* and other letters can be used as well. *f(x)* tells us that the result will be "a function of *x*", or is dependent upon *x*. The input, or independent variable, is *x* and the output, or dependent variable, is *f(x)* or y. Realize that *y = 2x + 5* and *f(x) = 2x + 5* are basically the same statement. The *f(x)* notation should be thought of as another way of representing the y-value, especially when graphing. The y-axis may be labeled as the *f(x)* axis.

**INTERPRETING FUNCTIONS**

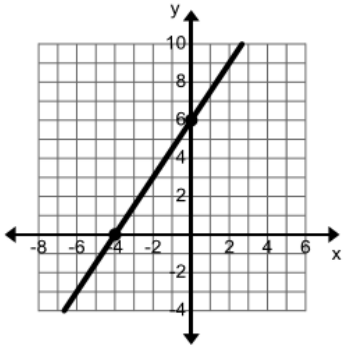
When analyzing a linear or exponential function within the context of a real application, you should be interpreting the intercepts, end behaviors, rates of change, domain, range, etc. within the context of function.

**FINDING INTERCEPTS**

The **x-intercept** is the point where the graph of a function touches or crosses the x-axis. It is of the form (a, 0) and can be found by substituting 0 in for y and solving for x. The **y-intercept** is the point where the graph of a function touches or crosses the y-axis. It is of the form (0, b) and can be found by substituting 0 in for x and solving for y.

**Example:** Find the x- and y-intercepts of the function 3x – 2y = -12

**Solution:**

| ***x*-intercept** | ***y*-intercept** | **Graph** |
|---|---|---|
| Let $y=0$.<br><br>$3x-2(0)=-12$<br>$3x-0=-12$<br>$3x=-12$<br>$x=-4$<br><br>The point $(-4,0)$ is the *x*-intercept. | Let $x=0$.<br><br>$3(0)-2y=-12$<br>$0-2y=-12$<br>$-2y=-12$<br>$y=6$<br><br>The point $(0,6)$ is the *y*-intercept. |  |



Sample Questions:

Find the x- and y-intercepts of the following functions:

1. 4x + 5y = 20

2. 2x + y = 2

3. 2x - y = -4

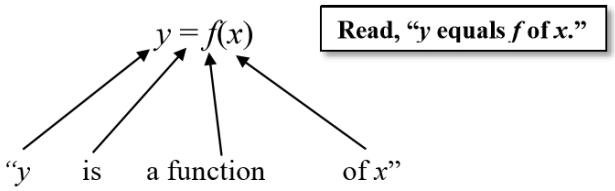
4. x - y = -5

**THE CONCEPT OF FUNCTIONS**

The concept of a function is both a different way of thinking about equations and a different way of notating equations. Let's look at a linear relationship the "old" way and with function notation.

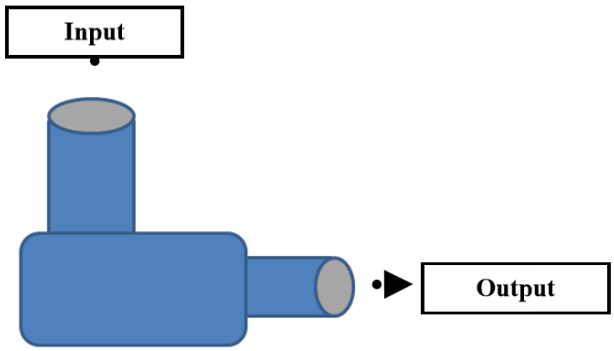
**Linear relationship: y = 2x + 1**

We say that *x* is the *independent variable* and *y* is the *dependent variable*. We choose values of x (the input) and then see what we get for y (the output). Another way to say this is: "The value of y is a function of (or depends on) our choice for x." When the above statement is written using mathematical symbols, it becomes:




**Note:** *y and f(x) may be used interchangeably; they both mean "what you get back after you have chosen a particular x value."*

One way to get a better grasp of the concept of a function is to picture the function as a machine. Our function machine is like a "**cause and effect map**" which typically shows the relationship between a cause and its effect. We will use a modified version to show the relationship between the input and output values in our "function machine." The domain values of x are the input (cause) and the range values of *f(x)* are the output (effect). Let's look at the function ***f(x) = -2x + 1***. When we input 0 for *x* (the cause), our output for *f(x)* is 1 (the effect).




| Input $x$ | $f(x) = -2x + 1$ | Output $f(x)$ |
|---|---|---|
| -2 | | |
| -1 | $f(-1) = -2(-1) + 1$ | 3 |
| 0 | $f(0) = -2(0) + 1$ | 1 |
| 1 | | |
| 2 | | |

**Example:** $f(x) = 2x^2 + 1$

**Solution:**

| | |
|---|---|
| We pick random values for $x$, substitute them into the equation for $x$, and simplify.<br>The numbers that we pick are the input values.<br>Let's pick $f(-1)$, $f(0)$ and $f(2)$.<br>We will use the same function each time. | $f(x) = 2x^2 + 1$ **Original Equation**<br>$f(0) = 2(0)^2 + 1$ **Substitute (0) for $x$**<br>$f(0) = 1$ **Solve** |
| $f(x) = 2x^2 + 1$ **Original Equation**<br>$f(-1) = 2(-1)^2 + 1$ **Substitute (-1) for $x$**<br>$f(-1) = 3$ **Solve** | $f(x) = 2x^2 + 1$ **Original Equation**<br>$f(2) = 2(2)^2 + 1$ **Substitute (2) for $x$**<br>$f(2) = 9$ **Solve** |

**Example:** *f(x) = 3x - 4*
**Solution:**

| | |
|---|---|
| We pick random values for $x$, substitute them into the equation for $x$, and simplify.<br>The numbers that we pick are the input values.<br>Let's pick $f(-1)$, $f(0)$ and $f(2)$.<br>We will use the same function each time. | $f(x) = 3x - 4$ **Original Equation**<br>$f(0) = 3(0) - 4$ **Substitute (0) for $x$**<br>$f(0) = -4$ **Solve** |
| $f(x) = 3x - 4$ **Original Equation**<br>$f(-1) = 3(-1) - 4$ **Substitute (-1) for $x$**<br>$f(-1) = -7$ **Solve** | $f(x) = 3x - 4$ **Original Equation**<br>$f(2) = 3(2) - 4$ **Substitute (2) for x**<br>$f(2) = 2$ **Solve** |

Sample Questions:
If *f(x) = 3x – 2*, find each value:

5. f(4)
6. f(-2)
7. f(2.5)
8. f(1/3)
9. f(-2/3)
10. f(2g)
11. f(p – 3)

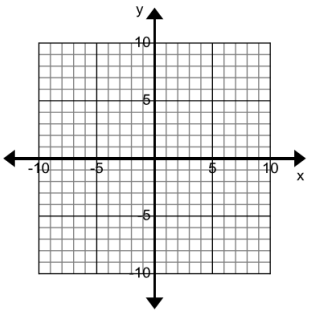
If $f(x) = \frac{x^2+2}{x-3}$ and $g(x) = x - x^2$, find each value:

12. f(2)
13. f(-2.5)
14. g(3)
15. f(a)
16. g(c – 3)

The next step is to solve for several points and graph the solution.

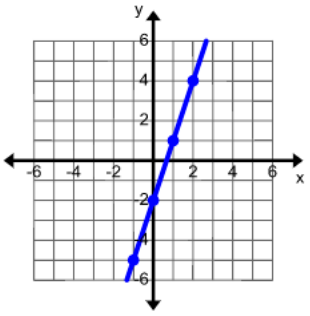
**Example:** Complete the table and graph the function
*f(x) = 3x - 2*

| $x$ | $f(x) = 3x - 2$ | $f(x)$ | $(x, f(x))$ |
|---|---|---|---|
| -2 | $f(-2) = 3(-2) - 2$ | $f(-2) = -8$ | (-2, -8) |
| -1 | | | |
| 0 | | | |
| 1 | | | |
| 2 | | | |




**Solution:**

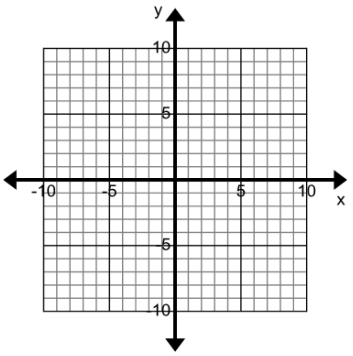
| $x$ | $f(x) = 3x - 2$ | $f(x)$ | $(x, f(x))$ |
|---|---|---|---|
| -2 | $f(-2) = 3(-2) - 2$ | -8 | $(-2, -8)$ |
| -1 | $f(-1) = 3(-1) - 2$ | -5 | $(-1, -5)$ |
| 0 | $f(0) = 3(0) - 2$ | -2 | $(0, -2)$ |
| 1 | $f(1) = 3(1) - 2$ | 1 | $(1, 1)$ |
| 2 | $f(2) = 3(2) - 2$ | 4 | $(2, 4)$ |




Sample Questions:
For the following problems, complete the table and graph the function.

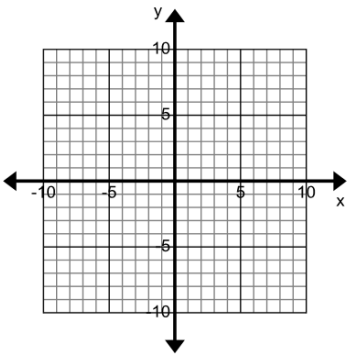
*17.* $f(x) = \frac{-2}{3}x + 1$

| $x$ | $f(x) = \frac{-2}{3}x + 1$ | $f(x)$ | $(x, f(x))$ |
|---|---|---|---|
| -2 | | | |
| -1 | | | |
| 0 | | | |
| 1 | | | |
| 2 | | | |




18. *f(x) = -2x + 1*

| $x$ | $f(x) = -2x + 1$ | $f(x)$ | $(x, f(x))$ |
|---|---|---|---|
| -2 | | | |
| -1 | | | |
| 0 | | | |
| 1 | | | |
| 2 | | | |



**DEPENDENT AND INDEPENDENT VARIABLES**

A variable that depends on one or more other variables. For equations such as y = 3x – 2, the dependent variable is y. The value of y depends on the value chosen for x. Usually the dependent variable is isolated on one side of an equation. Formally, a dependent variable is a variable in an expression, equation, or function that has its value determined by the choice of value(s) of other variable(s). The **in**dependent variable is the **in**put and the dependent variable is the output in the function scenario. The "in" is "in."

**WRITING FUNCTIONS FROM DATA**

In each of the examples above, the solutions were found from the function. However, functions can be derived from data as well. Remember the slope-intercept formula from coordinate geometry? This equation for linear lines works for linear functions as well.

$$y = mx + b$$

where m is the slope and b is the y intercept. First, use two points to calculate the slope or the rate of change of the function. Slope is the rise over the run or m = $\frac{y_2-y_1}{x_2-x_1}$. Then substitute one of the points into the equation for x and y and solve for b. See coordinate geometry for more practice. We can write the resulting equation as:

$$f(x) = mx + b$$

**Example:** Write an equation from the following data

| Input | Output |
|---|---|
| 3 | 7 |
| 5 | 11 |

**Solution A**: This is the same thing as finding the equation of a line through the points (3,7) and (5,11). First, calculate the slope from 2 points:

$$slope = \frac{y_2 - y_1}{x_2 - x_1} = \frac{11-7}{5-3} = \frac{4}{2} = 2$$

Then substitute the slope and either point into the slope-intercept formula and solve for the y-intercept (b).

$$\begin{aligned} 7 &= 2(3) + b \\ 7 &= 6 + b \\ -6 \quad &-6 \\ \hline 1 &= b \end{aligned}$$

Then substitute the slope and y-intercept into the slope-intercept formula to get

y = 2x + 1

We can also write an equation using the Point-Slope Formula:

$$y - y_1 = m(x - x_1)$$

where *m* is the slope and $(x_1, y_1)$ is a point on the line. This formula is sometimes more useful than the slope-intercept form in that you do not need the y-intercept, just a point on the line and the slope to create an equation of the line.

**Solution B:** Find the slope as above, but then use one of the data points and input into the point-slope formula.

| using (3,7): | or | using (5,11): |
|---|---|---|
| y – 7 = 2(x– 3) | | y – 11 = 2(x – 5) |

Each of these three equations are valid equations for the data set.

**FUNCTION WORD PROBLEMS**

There are many real-world problems that can be figured out using linear functions. To solve, first write an equation that represents the situation, then plug in the known (input) values and solve for the unknown (output) values. The information can be viewed in a table or as a graph.

**Example:** Bathtub Problem. You pull out the plug from the bathtub. After 40 seconds, there are 13 gallons of water left in the tub. One minute after you pull the plug, there are 10 gallons left. Assume that the number of gallons varies linearly with the time since the plug was pulled.

(a). Write a function expressing the number of gallons (g) left in the tub in terms of the number of seconds (s) since you pulled the plug.

(b). How many gallons would be left after 20 seconds? 50 seconds?

(c). Plot the graph of this linear function. Use a suitable domain.

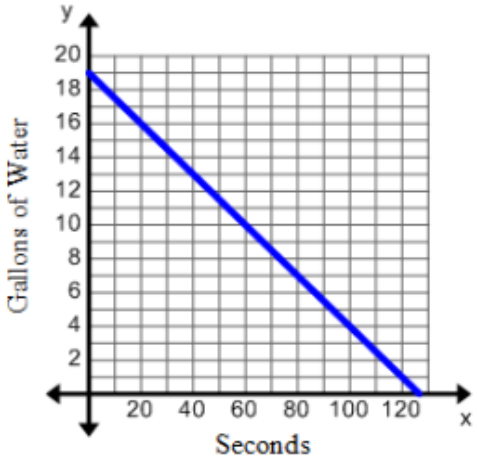
**Solution:** To solve (a) first, determine which variable is independent and which is dependent.
The Independent variable is the time which will be graphed on the x axis and the dependent variable is gallons of water left in the tub which will be graphed on the y-axis.
Next, from the data sets, calculate the slope and y-intercept.
The data sets are (40, 13) and (60, 10).
The slope is m = $\frac{13-10}{60-20}$ =- $\frac{3}{20}$
Substituting the point (60, 10) into the slope-intercept equation, the y-intercept is 10 = - $\frac{3}{20}$(60) – b
10 = -9 – b
+9 +9
19 = -b When both sides are divided by -1, b = 19.
Therefore, the function can be written as

$$g(s) = -\frac{3}{20}s + 19$$

(g because the dependent variable is gallons and s because the independent variable is seconds).

To solve for (b), plug in 20 and 50 into the equation found in part (a):
g(20) = -$\frac{3}{20}$(20) + 19 = 16 gallons
g(50) = -$\frac{3}{20}$(50) + 19 = 11.5 gallons

To solve for (c), plug in the data points already known on a graph.
(40, 13), (60, 10), (20, 16), and (50, 11.5) and draw a line through all points



Sample Questions:

19. Driving Home Problem: As you drive home from the football game, the number of kilometers you are away from home depends on the number of minutes you have been driving. Assume that the distance varies linearly with time. Suppose you are 11 km from home when you have been driving for 10 minutes, and 8 km from home when you have been driving for 15 minutes.

    a. Write a function expressing the number of kilometers you are from home (d) in terms of the number of minutes since you left the game (t).
    b. Predict your distance from home after driving for 20 min., 25 min., and 30 min.
    c. Plot the graph of this linear function. Use a suitable domain

20. Jane is tracking the progress of her plant's growth. Today the plant is 5 cm high. The plant grows .5 cm per day.
    a. Write a linear model that represents the height of the plant after d days.
    b. What will the height of the plant be after 20 days?

21. A salesperson receives a base salary of $35,000 and a commission of 10% of the total sales for the year.
    a. Write a linear model that shows the salesperson's total income based on the total sales of k dollars.
    b. If the salesperson sells $250,000 worth of merchandise, what is her total income for the year, including her base salary?

In the following sample questions,

a. Graph the relationship. Be certain to label the axes.
b. Record the following next to the graph: y-intercept, rate of change, and function equation

22. A toy company sold 17,000 stuffed animals in 1995. They have seen an increase of 900 stuffed animals per year.

23. Renting a canoe costs $10 for paperwork. There is an additional fee of $28 per day.

24. The temperature at the Earth's surface is 24℃. The temperature within Earth's crust increases about 30℃ for each kilometer beneath the surface.

25. The Spanish club website currently receives 500 daily visits. Daily visits are increasing by 20 each month.

26. A baby narwhal is born at 1 meter in length and grows ½ meter in length each year for about 8 years.

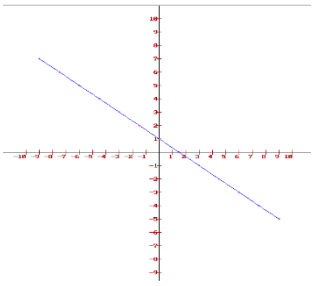
27. Sam has $27. At the end of every week, Sam saves another $12.50.

28. Suppose you have a piece of cake that makes you grow 7 inches taller every day. (Hint: What is your height in inches today?)

# Answer Key

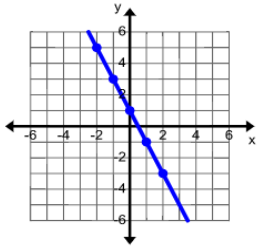
1. y intercept: (0, 4) x intercept: (5, 0)
2. y intercept: (0, 2) x intercept: (1, 0)
3. y intercept: (0, 4) x intercept: (-2, 0)
4. y intercept: (0, -5) x intercept: (5, 0)
5. 10
6. -8
7. 5.5
8. -1
9. -4
10. 6g – 2
11. 3p – 11
12. -6
13. -3/2
14. -6
15. $\frac{a^2+2}{a-3}$
16. $-c^2 + 7x - 12$
17.

| x | f(x)=-2/3x+1 | f(x) | (x, f(x)) |
|---|---|---|---|
| -2 | f(-2)=-2/3(-2)+1 | - 2/3 | (-2, 2 1/3) |
| -1 | f(-1)=-2/3(-1)+1 | -1 1/3 | (-1, 1 2/3) |
| 0 | f(0)=-2/3(0)+1 | -2 | (0, 1) |
| 1 | f(1)=-2/3(1)+1 | -2 2/3 | (1, 1/3) |
| 2 | f(2)=-2/3(2)+1 | -3 1/3 | (2, -1/3) |

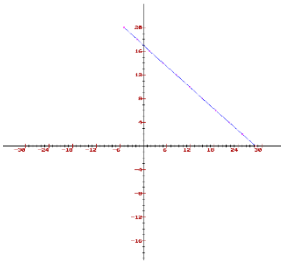


18.

| $x$ | | $f(x)=-2x+1$ | $f(x)$ | $(x,f(x))$ |
|---|---|---|---|---|
| -2 | | $f(-2)=-2(-2)+1$ | 5 | $(-2,5)$ |
| -1 | | $f(-1)=-2(-1)+1$ | 3 | $(-1,3)$ |
| 0 | | $f(0)=-2(0)+1$ | 1 | $(0,1)$ |
| 1 | | $f(1)=-2(1)+1$ | -1 | $(1,-1)$ |
| 2 | | $f(2)=-2(2)+1$ | -3 | $(2,-3)$ |

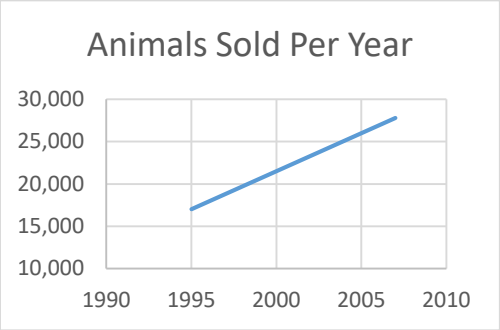



19. A d(t) = – 3/5t + 17
    B 20 m = 5 km 25 min = 2 km 30 min = been home for 1 min
    C

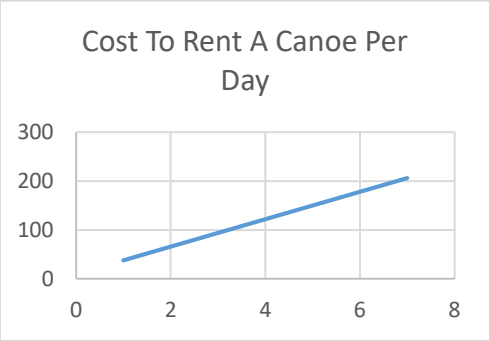


20. A h(d) = 1/2d + 5
    B 15 cm
21. A I(k) = 0.1k + 35,000
    B $60,000

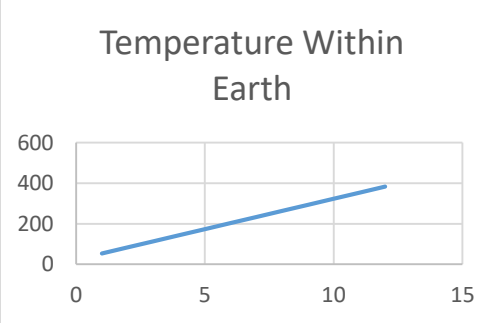
22. B = 17,000  m = 900
A(y) = 900Y + 17,000




23. B = 10  m = 28
C(d) = 28D + 10

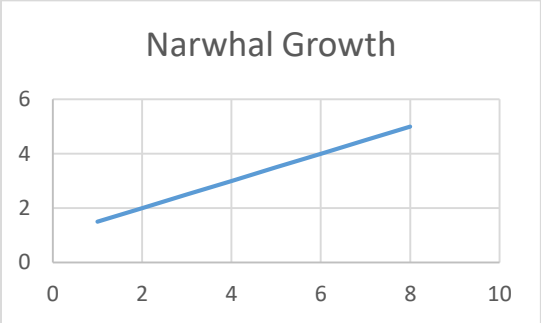



24. B = 24  m = 30
T(d) = 30D + 24

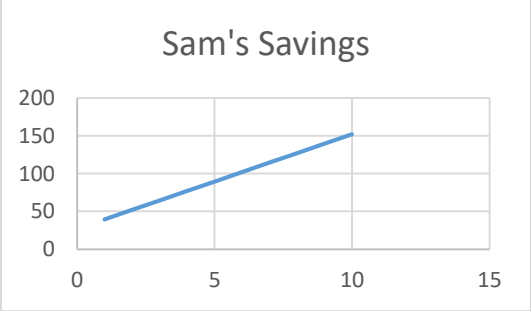


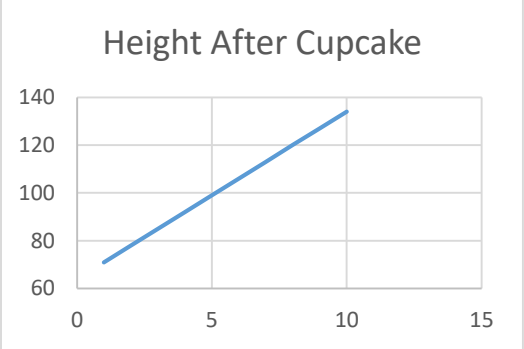

25. B = 300  m = 20
V(m) = 20m + 500

26. B = 1  m = 0.5
G(y) = 0.5(y) + 1




27. B = $27  m = $12.50
S(w) = $12.50w + $27




28. B = 64 (any reasonable number)  m = 7
H(d) = 7d + 64 (any reasonable number)