Math 1 Variable Manipulation Part 4
Word Problems

TRANSLATING FROM ENGLISH INTO ALGEBRA (PLUG IN)

The next part of variable manipulation problems is to figure out the problem from real life situations. These problems have a lot of wording to help explain the problem in English format. Some of the problems will have the equation within the explanation. There will usually be an explanation of the variables and the value of the variables.

Example: While doing research on the climates of South American countries, Andrea notices that all of the temperatures are given in degrees Celsius. Because she is not as familiar with Celsius temperature scale, it is difficult for her to know whether a location with an average temperature of 25℃ has a warm climate. Fahrenheit, ℉, and Celsius, ℃, are related by the formula $F = (\frac{9}{5})\ C + 32$. What is the temperature in degrees Fahrenheit of the location with an average temperature 25℃?

Solution: The equation for this problem is $F = (\frac{9}{5})\ C + 32$ as listed within the problem. The problem also states that C is the degrees Celsius and says to find the degrees F when degrees C is equal to 25? So plug in 25 for C and solve for F.

$$F = (\frac{9}{5})\ C + 32 = (\frac{9}{5})\ 25 + 32 = 45 + 32 = 77$$

Sample Questions:

1. The length L, in meters, of a spring is given by the equation $L = (2/3)F + 0.05$, where F is the applied force in newtons. Approximately what force, in newtons, must be applied for the spring's length to be 0.23 meters?

2. A formula for calculating simple interest is $I = Pr$, where I is the interest earned in dollars, P is the principal or original investment, and r is the fixed rate of interest. If the amount of interest earned is \$2.25 and the interest rate is 3%, what is P?

3. A formula used to compute the current value of an investment account is $A = P(1 + r)^n$, where A is the current value, P is the amount deposited, r is the rate of interest for 1 compounding period, expressed as a decimal, and n is the number of compounding periods. What is the value of an investment account after 3 years if \$8,000 is deposited at 5% annual interest compounded annually?

4. Molality, m, tells us the number of moles of solute dissolved in exactly 1 kilogram (kg) of solvent. Molality is represented by the equation, $m = \frac{s}{k}$, where s represents the moles of solute and k represents the mass of the solvent in kilograms. A solution is known to have a molality of 0.2 and contain 13 kg of solvent. What is the number of moles of solute contained in the solution?

5. For a population that grows at a constant rate of r% per year, the formula $P(t) = p_0(1 + \frac{r}{100})^t$ models the population t years after an initial population of p_0 people is counted. The population of the city of San Jose was 782,000 in 1990. Assume the population grows at a constant rate of 5% per year. According to this formula, which of the following is an expression for the population of San Jose in the year 2000?
   a. $782{,}000(6)^{10}$
   b. $782{,}000(1.5)^{10}$
   c. $782{,}000(1.05)^{10}$
   d. $(782{,}000 \times 1.5)^{10}$
   e. $(782{,}000 \times 1.05)^{10}$

6. Force is related to mass and acceleration by the equation F = m * a where F = force exerted, m = mass of the object and a = acceleration of the object. If worker drops his hammer off the roof of a house, how much force does the earth apply to the hammer when it hits the ground? The hammer has a mass of 9 kg, and gravity accelerates it at the usual 9.8 m/s^2.
   a. 40 $kg*m/s^2$
   b. 56 $kg*m/s^2$
   c. 67 $kg*m/s^2$
   d. 79 $kg*m/s^2$
   e. 88 $kg*m/s^2$

TRANSLATING FROM ENGLISH INTO ALGEBRA (EQUATIONS)

To translate from English into algebra, look for the key words and work from left to right to turn phrases into algebraic expressions and sentences into equations. Be careful about order, especially when subtraction is called for.

Translating Verbal Phrases	
Addition	Sum, plus, total, more than, increased by
Subtraction	Difference, less than, minus, decreased by
Multiplication	Times, product, multiplied by, of twice, double, triple, etc.
Division	Quotient, divided by, divided into, half, one-fourth, etc.
Equals	Is, the same as

Example: For all real numbers *b* and *c* such that the sum of *c* and 10 is *b*, which of the following expressions represents the product of *c* and 3 in terms of *b?*

a. b+ 10
b. 3b - 30
c. 3(b + 3)
d. $\frac{b+3}{3}$
e. $\frac{b}{3}$ + 3

Solution: The sum of c and 10 is b is translation by knowing that sum means addition and is means equal to. The equation for the first phrase is written as c + 10 = b. Solving for c becomes c = b - 10. Substitute that for c into the second equation and get c x 3 = (b-10) x 3 = 3b -30 or B.

Sample Questions:

7. What is the sum of 20 decreased by 17 and the product of 2 and 3?

8. Write an equation equal to the sum of a number and eight equals six squared.

9. What is the difference between half of 120 and the product of 4 and 2.

10. What is 14 less than the product of 25 and 4?

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11. If each element in a data set is multiplied by 3, and each resulting product is then reduced by 4, which of the following expressions gives the resulting data set in terms of x?
    a. x
    b. 3x - 4
    c. x + $\frac{4}{3}$
    d. $\frac{x}{3}$ + 4
    e. x + $\frac{3}{4}$

12. Which of the following mathematical expressions is equivalent to the verbal expression "A number, *x*, squared is 39 more than the product of 10 and *x*"?
    a. $x^2 = 39 \times 10x$
    b. $x^2 = 39 + 10x$
    c. $x^2 = 390 - 10x$
    d. $x^2 = 39x + 10$
    e. $x^2 = 390 + 10x$

TRANSLATING FROM ENGLISH INTO ALGEBRA (STORY PROBLEMS)

Equations can be used to solve many real life problems. To write an equation that matches a situation translate from English words into algebra, look for the key words and work from left to right to turn phrases into algebraic expressions. Identify the unknown for which you are looking for and assign a variable to it. Be careful about order, especially when subtraction is called for.

Example: The charge for a phone call is r cents for the first 3 minutes and s cents for each minute thereafter. What is the cost, in cents, of a call lasting exactly t minutes? ($t > 3$)

Solution: The charge begins with r, and then something more is added, depending on the length of the call. The amount added is s times the number of minutes past 3 minutes. If the total number of minutes is t, then the number of minutes past 3 is $t - 3$. So the charge is $r + s(t- 3)$.

Three steps to help solve Word Problems

1. **Know the question.** Read the whole problem before calculating anything and underline important info.
2. **Let the answers help**. Look for clues on how to solve and ways to use process of elimination (POE)
3. **Break the problem into bite-sized pieces.** When you read the problem a second time, calculate at each step necessary and watch out for tricky phrasing.

Example: Sandy had 102 dollars to spend on 7 books. After buying them she had 18 dollars. How much did each book cost?

Solution: The fact that Sandy had \$102 to spend on books sounds like that would be the total. Except when reading further on, she had 18 left. So, Sandy actually spent \$102 - \$18. She bought 7 books and we are trying to figure out how much each book costs. So, the cost would be x and the amount spent would be 7x.

So, the equation would be: $7x = 102 – 18$

Since the question asked for how much each book will cost, we solve for x to get the answer.

Solving for x, $\frac{102-18}{7} = 12$

Sample Questions:

13. The fixed costs of printing a certain textbook are \$900.00 per week. The variable costs are \$1.50 per textbook. Which of the following expressions can be used to model the cost of printing t textbooks in 1 week?
    a. \$901.50t
    b. \$150t - \$900.00
    c. \$900.00t + \$1.50
    d. \$900.00 - \$1.50t
    e. \$900.00 + \$1.50t

14. Two enterprising college students decide to start a business. They will make up and deliver helium balloon bouquets for special occasions. It will cost them \$39.99 to buy a machine to fill the balloons with helium. They estimate that it will cost them \$2.00 to buy the balloons, helium, and ribbons needed to make each balloon bouquet. Which of the following expressions could be used to model the total cost for producing *b* balloon bouquets?
    a. \$ 2.00*b* + \$39.99
    b. \$37.99*b*
    c. \$39.99*b* + \$ 2.00
    d. \$41.99*b*
    e. \$79.98*b*

15. The fixed costs of manufacturing basketballs in a factory are $1,400.00 per day. The variable costs are $5.25 per basketball. Which of the following expressions can be used to model the cost of manufacturing b basketballs in 1 day?
    a. $1,405.25b
    b. $5.25b - $1,400.00
    c. $1,400.00b + $5.25
    d. $1,400.00 - $5.25b
    e. $1,400.00 + $5.25b

16. Leticia went into Discount Music to price CDs. All CDs were discounted 23% off the marked price. Leticia wanted to program her calculator so she could input the marked price and the discounted price would be the output. Which of the following is an expression for the discounted price on a marked price of *p* dollars?
    a. $p - 0.23p$
    b. $p - 0.23$
    c. $p - 23p$
    d. $p - 23$
    e. $0.23p$

17. A house painter charges $24.00 per hour for a painting job that requires more than 5 hours to complete. For any job requiring 5 hours or less, the painter charges a flat fee of $100. If n represents the number of hours the job requires, which of the following expressions gives the charge, in dollars, for a job requiring more than 5 hours to complete?
    a. 124.0
    b. -24n + 100
    c. 24n – 100
    d. 24n
    e. 24n + 100

TRANSLATING FROM ENGLISH INTO ALGEBRA (SOLVE FOR X)

To translate from English into algebra, look for the key words and work from left to right to turn phrases into algebraic expressions and sentences into equations. Find the important numbers and write an equation that makes sense. Find out what answer the question is asking and solve your equation for the answer (solve for x) by using the order of operation rules (PEDMAS).

Example: The membership fees for WebFilms consists of a monthly charge of $14 and a one-time new-member fee of $16. Sherwood made a credit card payment of $100 to pay his WebFilms fees for a certain number of months, including the new-member fee. How many months of membership did Sherwood include in his credit card payment?
    a. 4
    b. 6
    c. 7
    d. 12
    e. 14

Solution: Sherwood's $100 payment covers both his one-time new-member fee and a few months of a membership. Since the new-member fee is $16 and the monthly fee is $14, put (14 x m) into the equation. The equation would be written as 14m + 16 = 100. Solve for the number of months (m) by subtracting 16 from each side of the equation to get 14 m = 84. Divide both sides of the equation by 14 to get m = 6 or B.

Sometimes ACT problems have lots of words and complicated equations within the questions. When you see one of these type of problems, get excited because while these problems look complicated, they are usually very easy to solve.

Example: A formula for finding the value, A dollars, of P dollars invested at i% interest compounded annually for n years is $A = P(1 + 0.01i)^n$. Which of the following is an expression for P in terms if I, n and A?

a. $A - 0.01i^n$
b. $A + 0.01i^n$
c. $\left(\frac{A}{1+0.01i}\right)^n$
d. $\frac{A}{(1-0.01i)^n}$
e. $\frac{A}{(1+0.01i)^n}$

Solution: The formula for this problem is listed within the text. It looks complicated, $A = P(1 + 0.01i)^n$, however, the problem is just asking to solve in terms of P. Since P is multiplied by a bunch of stuff, just divide both sides if the equation by that bunch of stuff to get P alone. $P = A / (1 + 0.01i)^n$ or E.

Sample Questions:

18. An editor charges $30 for each hour he works on a book project, plus a flat $25 editing fee. How many hours of work are included in a $190 bill for a book project?

19. On Monday, 217 students went on a trip to the zoo. All 4 buses were filled and 9 students had to travel in cars. How many students were in each bus?

20. Alyssa sold half of her comic books and then bought 7 more. She now has 18. How many did she begin with?

21. Oceanside Bike Rental Shop charges 18 dollars plus 7 dollars an hour for renting a bike. Sally paid 60 dollars to rent a bike. How many hours did she pay to have the bike checked out?

22. Two professors were hired to begin work at the same time. Professor A's contract called for a starting salary of $50,000 with an increase of $1,500 after each year of employment. Professor B's contract called for a starting salary of $42,000 with an increase of $2,800 after each year of employment. If y represents the number of full years of employment (that is, the number of yearly increases each professor has received,) determine the number of years until b's yearly salary equals A's yearly salary?

Answer Key

1. 0.27
2. $75.00
3. $9,261
4. 2.6
5. C
6. 88 kg*m/s^2
7. 9
8. N + 8 = 6^2
9. 52
10. 86
11. 3x – 4
12. B
13. E
14. A
15. E
16. A
17. D
18. 5 ½
19. 52 students
20. 22 comic books
21. 6 hours
22. 7 years