Name:_____ Date:_____

Math 1 Variable Manipulation Part 6
Polynomials

VOCABULARY

Constant: A term that does not have a variable is called a constant.
Example: the number 5 is a constant because it does not have a variable attached and will always have the value of 5.

Term: A constant or a variable or a product of a constant and a variable is called a term.
Example: 2, x, or $3x^2$ are all terms.

Polynomial: An expression formed by adding a finite number of unlike terms is called a polynomial. The variables can only be raised to positive integer exponents. Note that there are no square roots of variables, no fractional powers and no variables in the denominator of any fractions.
Example A: $4x^3 – 6x^2 + 1$ is a polynomial
Example B: $x^{½} - 2x^{-1} + 5$ is NOT a polynomial

Monomial, Binomial, Trinomial: a polynomial with only one term is called a monomial. A polynomial with two terms is called a binomial and a polynomial with three terms is a trinomial.
Examples:
$6x^4$ (monomial)
$5x + 1$ (binomial)
$2x^2 – x + 3$ (trinomial)

Standard (General) Form: Polynomials are in standard form when written with exponents in descending order and the constant term is last.
Example: $2x^4 – 5x^3 + 7x^2 –x +3$ is in standard form

Degree: The exponent of a term gives you the degree of the term. For a polynomial, the value of the largest exponent is the degree of the whole polynomial.
Example A: $-3x^2$ has degree two
Example B: $2x^4 – 5x^3 + 7x^2 –x + 3$ has a degree four

Coefficient: The number part of a term is called the coefficient when the term contains a variable and a number.
Example A: 5x has a coefficient of 5
Example B: -x2 has a coefficient of -1

Leading coefficient: The leading coefficient is the coefficient of the first term when the polynomial is written in standard form.
Example: The leading coefficient of $2x^4 – 5x^3 + 7x^2 –x + 3$ is 2

General Polynomial:
$a_nx^n + a_{n-1} x^{n-1} + . . . a_2x^2 + a_1x + a_0$
where a_n is the Leading Coefficient, a_nx^n is the leading term, the degree is n and a_0 is a conatant.

ADDING AND SUBTRACTING POLYNOMIALS

Polynomials is a word that describes a bunch of terms added to or subtracted from each other. A monomial has just one term, a binomial has two terms and a trinomial has three terms. A polynomial has any number of terms. Adding or subtracting polynomials is just combining like terms. If parentheses separate some terms from other terms, then make sure to distribute the + or the – into all the terms inside the parentheses.

Example: $(3x^2 + 5x - 7) - (x^2 + 12) = ?$

Solution: Add up the number of x^2, x and constants separately.

$(3x^2 + 5x - 7) - (x^2 + 12) =$

$(3x^2 - x^2) + 5x + (-7 - 12) =$

$2x^2 + 5x - 19$

Sample Questions:

1. $(6a – 12) – (4a + 4) = ?$

2. $x^2 + 60x + 54 – 59x – 82x^2$ is equivalent to:
   a. $-26x^2$
   b. $26x^6$
   c. $-81x^2 + x + 54$
   d. $81x^2 + x + 54$
   e. $-83x^2 –x -54$

3. $(4x – 3x^3) –(3x^3 + 4x)$

4. $(3a^2 + 1) – (4 + 2a^2) = ?$

5. $(5a + 4) – (5a + 3) = ?$

6. $(-4k^4 + 14 + 3k^2) + (-3k^4 – 14k^2 – 8) = ?$

MULTIPLYING MONOMIALS

To multiply monomials, multiply the numbers and the variables separately. The numbers are multiplied as normal and the exponents of the variables are added together.

Example: 2a x 3a = ?
Solution: 2a x 3a = (2 x 3)(a x a) = $6a^2$

Multiplying a monomial by a binomial is the same as distributing the monomial into the binomial.
Example: 2a(3a – 4) = ?
Solution: 2a*3a – 2a*4 = $6a^2$ – 8a

Sample Questions:

7. 7(-5x – 8)

8. 2x(-2x – 3)

9. For all x, x^2 – (3x – 2) + 2x(4x – 1) = ?

10. Which of the following is an equivalent form of x + x(x + x + x)?
    a. 5x
    b. x^2 + 3x
    c. $3x^2$ + x
    d. $5x^2$
    e. x^3 + x

11. For all x, (3x)(2) – 3x(2x + 4) + 5(x – 3) = ?

12. Which of the following is an equivalent form of 2x + 3x – 5 + 4 – x(3 + 4)
    a. 5x – 1 – $7x^2$
    b. 2x + 7
    c. -2x – 1
    d. 5x - 7
    e. 6x + 1

SOLVING "IN TERMS OF"

Some equations have more than one variable, more than one unknown. To solve an equation for one variable in terms of another means to isolate the one variable that you are solving for on one side of the equation, leaving an expression containing the other variable on the other side.

Example: Solve 3x - 1Oy = -5x + 6y for x in terms of y

Solution: Isolate x on one side and y on the other by adding 5x to both sides and adding 10y to both sides. Then divide both sides by 8.

3x - 10y = -5x + 6y

3x + 5x = 6y + 1Oy

8x = 16y

x= 2y

Sample Questions:

13. For all pairs of real numbers M and N where M = 6N + 5, N = ?

14. If the expression $x^3 + 2hx - 2$ is equal to 6 when x = -2, what is the value of h?

15. Solve for x in terms of y: 2x + 8y = 4

16. Solve for x in terms of y: 3x + 6y = 9

Equivalent Forms

Besides plugging in numbers to get an answer, solving in terms of x or another variable, there are ways to simplify equations and express them as equivalent equations. This can be accomplished in many ways. Equations will be equivalent as long as all mathematical rules are followed.

Example: Which expression below is equivalent to w(x – (y + z))?

a. wx – wy – wz
b. wx – wy + wz
c. wx – y + z
d. wx – y - z
e. wxy + wxz

Solution: To find an equivalent for the given expression, use the distributive property. First, evaluate the inner parentheses according to the order of operations, or PEMDAS. Distribute the negative sign to (y + z) to get w(x – y – z). Next, distribute the variable w to all terms in parentheses to get wx – wy – wz or A. Choice B fails to distribute the negative sign to the z term. Choices C and D only distribute the w to the first term. Choice E incorrectly distributes wx to the (y + z) term.

Sample Questions:

17. Which of the following is a simplified form of 4x – 4y + 3x?
    a. x(7 – 4y)
    b. x – y + 3x
    c. -8xy + 3x
    d. 7x – 4y
    e. -4y –x

18. $\frac{2r}{3} + \frac{4s}{5}$ is equivalent to:
    a. $\frac{2r+4s}{8}$
    b. $\frac{2r+4s}{15}$
    c. $\frac{2(r+2s)}{15}$
    d. $\frac{(10r+12s)}{15}$
    e. $\frac{2(10r+12s)}{15}$

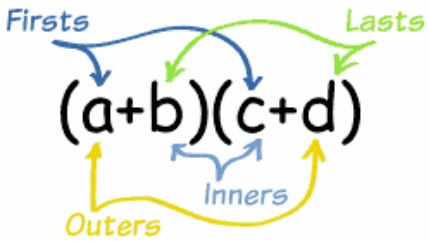
19. When $y = x^2$, which of the following expressions is equivalent to $-y$?
    a. $(-x)^2$
    b. $-x^2$
    c. $-x$
    d. x^{-2}
    e. x

20. Which of the following is always equal to y(3 – y) + 5(y – 7)?
    a. 8y -35
    b. 8y -7
    c. $-y^2 + 8y - 7$
    d. $-y^2 + 8y - 35$
    e. $8y^3 - 35$

21. If W = XYZ, then which of the following is an expression for Z in terms of W, X, and Y?
    a. $\frac{XY}{W}$
    b. $\frac{W}{XY}$
    c. WXY
    d. W – XY
    e. W + XY

MULTIPLYING BINOMIALS-FOIL

When multiplying two binomials, distribute both terms in the first binomial to both terms in the second binomial. We commonly call this the FOIL method. FOIL: First, Outer, Inner, Last.



Example: $(2x + 3)(4x – 5)$

Solution:

First multiply the First terms: $2x * 4x = 8x^2$

Next the Outer terms: $2x * -5 = -10x$

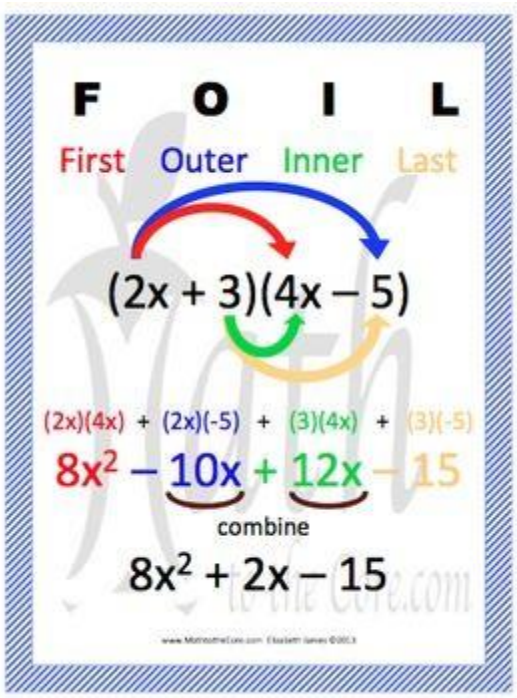
Then the Inner terms: $3 * 4x = 12x$

And finally the Last terms: $3 * -5 = -15$

Then add together and combine like terms:

$8x^2 - 10x + 12x - 15 = 8x^2 + 2x - 15$

Visual solution to the right →



Example: $(x + 3)(x + 4)$

Solution:

First multiply the First terms: $x * x = x^2$

Next the Outer terms: $x * 4 = 4x$.

Then the Inner terms: $3 * x = 3x$.

And finally the Last terms: $3 * 4 = 12$.

Then add together and combine like terms:

$x^2 + 4x + 3x + 12 = x^2 + 7x + 12$.

Sample Questions:

22. FOIL the following: $(x – 5)(2x + 3)$?

23. The expression $(2x – 3)(x + 8)$ is equivalent to:

24. FOIL the following: $(x + 6)(x + 4)$

25. Use the FOIL method for: $(x + 8)(x – 2)$

26. The expression $(3x - 3)(2x + 4)$ is equivalent to:

27. FOIL the following: $(3a + 6)(a – 2)$

28. The expression $(3x – 4y^2)(3x + 4y^2)$ is equivalent to:

29. The expression $(4z + 3)(z – 2)$ is equivalent to:

FACTORING OUT A COMMON TERM

A factor common to all terms of a polynomial can be factored out. All three terms in the polynomial $3x^3 + 12x^2 - 6x$ contain a factor of 3x. Pulling out the common factor yields $3x(x^2 + 4x - 2)$. Remember that if you factor a term out completely, you are still left with 1.

Example: $6x^2 + 9x + 3$

Solution: Factor a 3 out of everything. You're left with $3(2x^2 + 3x + 1)$.

Sample Questions:

30. $–x^2 – x + px$

31. $36x – 12x^2$

32. $3x^3 + 27x^2 + 9x$

FACTORING OTHER POLYNOMIALS-FOIL IN REVERSE

To factor a quadratic expression, think about what binomials you could use FOIL on to get that quadratic expression. To factor $x^2 - 5x + 6$, think about what First terms will produce x what Last terms will produce + 6, and what Outer and Inner terms will produce -5x. In other words, if there is no number in front of the first term, you are looking for two numbers that add up to the middle term and multiply to get the third term. So here, you'd want two numbers that add up to -5 and multiply to 6. (Pay attention to sign-negative vs. positive makes a big difference here!) The correct factors are (x - 2)(x - 3). You can also solve for x for each factor. In this case x = 2 and x = 3.

Note: Not all quadratic expressions can be factored. These expressions will be covered in Math 2.

Example: Factor the expression $x^2 + 5x + 6$

Solution: Since the first term is x^2, the first numbers in each binomial will be x. Next, find two numbers that multiply to be +6 and add to be +5.

The factors that multiply to +6 are:

1 & 6

-1 & -6

2 & 3

-2 & -3

The only factors that also add to +5 are 2 & 3. So the answer is

(x + 2)(x + 3)

Sample Questions:

33. Which of the following is a factored form of the expression $x^2 - 1x - 6$?
    a. $(x - 3)(x + 2)$
    b. $(x - 2)(x - 3)$
    c. $(x - 2)(x + 3)$
    d. $(x + 2)(x + 3)$
    e. $(x + 3)(x - 3)$

34. Factor $x^2 - 2x - 8$

35. Which of the following is a polynomial factor of $x^2 - 2x - 24$?
    a. x – 4
    b. x + 4
    c. x + 6
    d. 6 - x
    e. x

36. Factor $a^2 - a - 90$

37. Factor $p^2 + 11p + 10$

38. Factor $b^2 – 6b + 8$

FACTORING: SPECIAL CASES

Factoring the difference of squares: One of the test maker's favorite classic quadratics is the difference of squares.

$a^2 - b^2 = (a - b)(a + b)$

Example: $x^2 - 9$

Solution: $x^2 - 9$ factors to $(x - 3)(x + 3)$

Factoring the square of a binomial: There are two other classic quadratics that occur regularly on the ACT:

$a^2 + 2ab + b^2 = (a + b)^2$

$a^2 - 2ab + b^2 = (a - b)^2$

Example: $4x^2 + 12x + 9$

Solution: $4x^2 + 12x + 9$ factors to $(2x + 3)(2x + 3)$ which can be written $(2x + 3)^2$

Example: $n^2 - 10n + 25$

Solution: $n^2 - 10n + 25$ factors to $(n - 5)(n - 5)$ which can be written $(n - 5)^2$

Note: Recognizing a classic quadratic can save a lot of time on Test Day-be on the lookout for these patterns. Any time you have a quadratic and one of the numbers is a perfect square, you should check for one of these three patterns.

Sample Questions:

39. For all n, $(3n+ 5)^2 = ?$

40. For all x, $(3x + 7)^2 = ?$

41. $(\frac{1}{3}a – b)^2 = ?$

Answer Key

1. $2a - 16$
2. $-81x^2 + x + 54$
3. $-6x^3$
4. $a^2 - 3$
5. 1
6. $-7k^4 - 11k^2 + 6$
7. $-35x - 56$
8. $-4x^2 - 6x$
9. $9x^2 - 5x + 2$
10. $3x^2 + x$
11. $-6x^2 - x - 15$
12. $-2x - 1$
13. $N=(M-5)/6$
14. $x = -4$
15. $x = 2 - 4y$
16. $x = 3 - 2y$
17. $7x - 4y$
18. $\frac{(10r+12s)}{15}$
19. $-x^2$
20. D
21. B
22. $2x^2 - 7x - 15$
23. $2x^2 + 13x - 24$
24. $x^2 + 10x + 24$
25. $x^2 + 6x - 16$
26. $6x^2 + 6x - 12$
27. $3a^2 - 12$
28. $9x^2 - 16y^4$
29. $4z^2 - 5z - 6$
30. $-x(x + 1 - p)$
31. $12x(3 - x)$
32. $3x(x^2 + 9x + 3)$
33. $(x - 3)(x + 2)$
34. $(x + 2)$ and $(x - 4)$
35. $x + 4$
36. $(a - 10)(a + 9)$
37. $(p + 10)(p + 1)$
38. $(b - 4)(b - 2)$
39. $9n^2 + 30n + 25$
40. $9x^2 + 42x + 49$
41. $\frac{1}{9}a^2 - \frac{2}{3}ab + b^2$