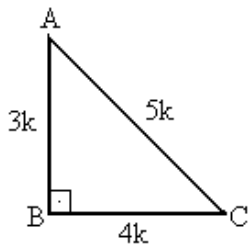
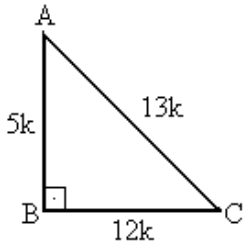
Name:_____ Date:_____

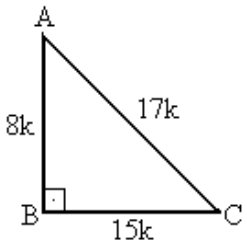
Math 1 Plane Geometry Part 3

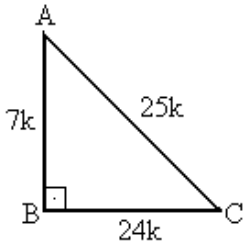
Special triangles

When using the Pythagorean theorem, we often get answers with square roots or long decimals. There are a few special right triangles that give integer answers. We've already talked about the 3-4-5 right triangle, now it's time to learn a few others that also give integer answers. When you are able to recognize these kinds of triangles, you don't even have to use the Pythagorean theorem, which makes things simpler and easier. If you forget or don't recognize these, no worries, just use the Pythagorean theorem, it works every time.

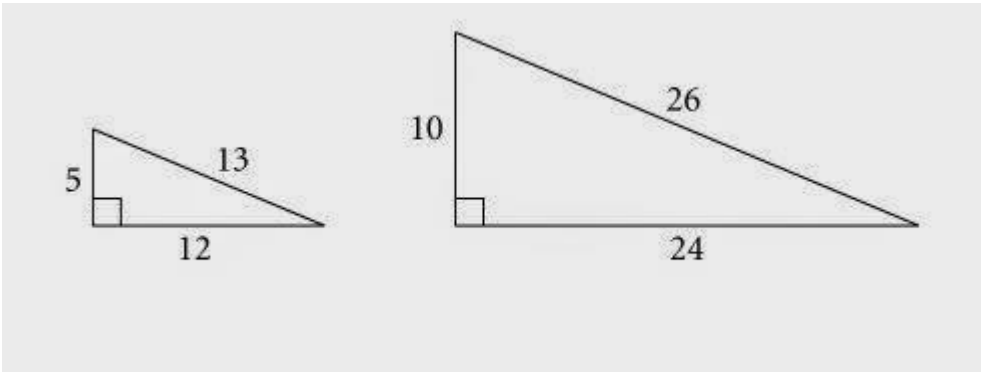




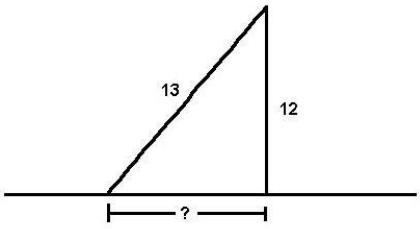




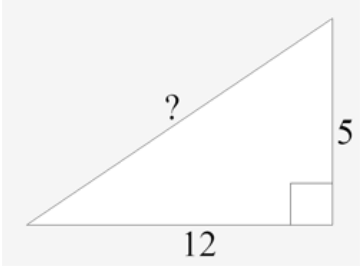
The special right triangles are the 3-4-5, the 5-12-13, the 8-15-17, and the 7-24-25. On the figures above the "k" by each number just means that multiples of these work as well. For example in the figures below, these are both 5-12-13 triangles. The one on the right is two times 5-12-13.



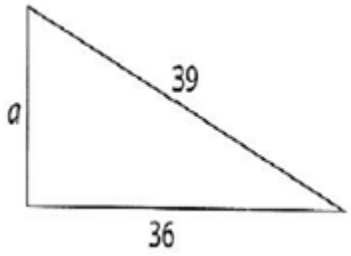
1. The figure to the right is a right triangle. What is the length of the base leg?



2. The figure below is a right triangle. What is the length of the hypotenuse?

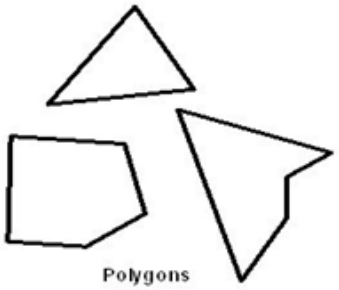


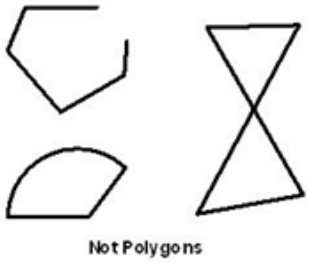
3. The figure below is a right triangle. Solve for a.



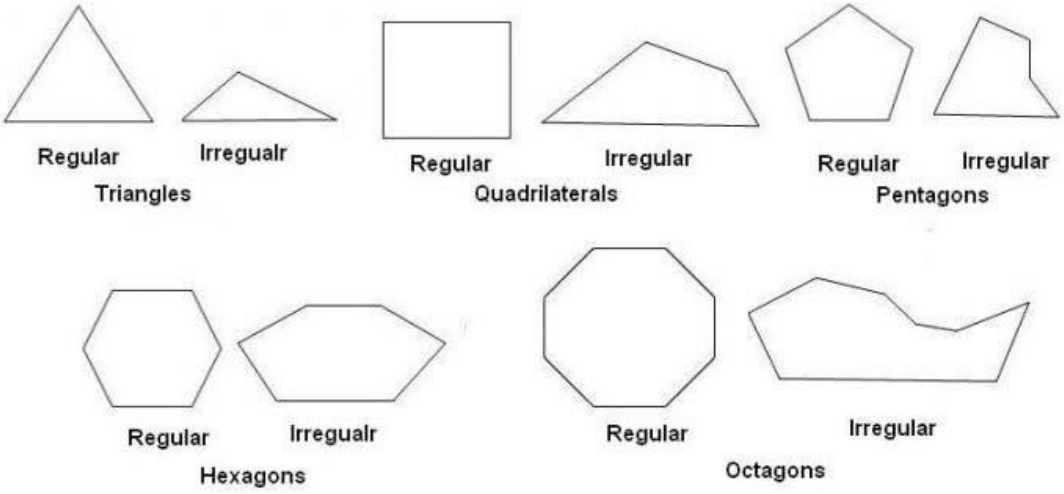
More on polygons.

The term "polygon" is a combination of two Greek words, "poly" which means "many" and "gonia" which means "angle." A polygon is a two-dimensional closed figure bounded by straight line segments. Most of the basic shapes, such a triangles, squares, rectangles, etc. are examples of polygons. Circles are not polygons. They don't have straight sides or angles.

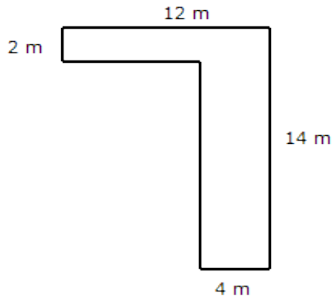




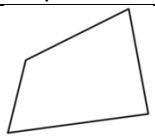
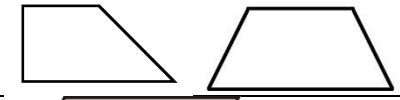
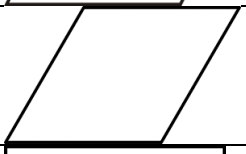
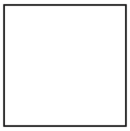
Polygons are named by how many sides or angles they have. Triangles have 3 sides, quadrilaterals have 4 sides, pentagons have 5 sides, etc. The shapes that we usually think of are called "regular" polygons, but polygons can be shaped irregularly as well.



4. Find the perimeter of the hexagon below.



A single shape may fit under many different categories. For example, a square is also a rectangle, and a parallelogram,and a rhombus, and a quadrilateral, and a polygon because it fits the definitions of each.

Shape	Characteristic	Name
	No sides parallel	Trapezium
	Exactly one pair of parallel sides	Trapezoid
	Two pairs of parallel sides	Parallelogram
	Parallelogram with congruent sides	Rhombus
	Parallelogram with right angles	Rectangle
	Rectangle with congruent sides	Square

Note that squares, rectangles, and rhombuses are types of parallelograms and that a square is a type of rectangle and a type of rhombus.

5. A square is a parallelogram

A. Always
B. Sometimes
C. Never

6. A rhombus is a rectangle

A. Always
B. Sometimes
C. Never

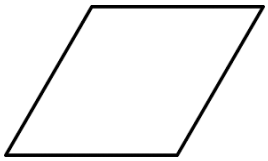
7. A parallelogram is a trapezoid

A. Always
B. Sometimes
C. Never

8. A rectangle is a square

A. Always
B. Sometimes
C. Never

9. The figure below bleongs in which of the following classifications?



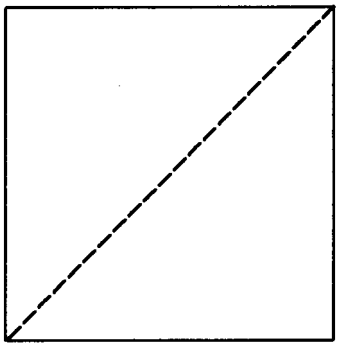
I. Polygon
II. Quadrilateral
III. Parallelogram
IV. Rhombus
V. Square

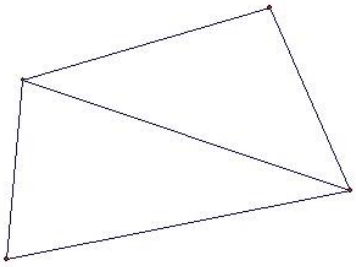
A. I, II, III only
B. V only
C. II, III, IV only
D. I, II, III, IV only
E. I, II, III, IV, and V

Interior angles of a polygon

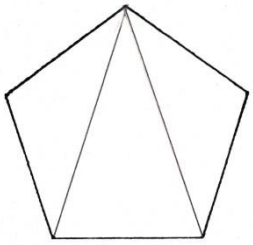
We know that the sum of the interior angles of a triangle equal 180°. We can use this information to find the sum of the interior angles of other polygons.

A square (or any quadrilateral) can be divided into 2 triangles. Since the interior angles of each of the triangles will be 180°, the total will be 360°.

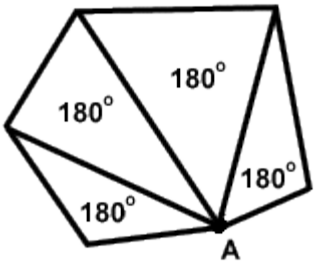




A pentagon can be divided into 3 triangles. Since the interior angles of each of the triangles will be 180°, the total for the pentagon will be 540°.



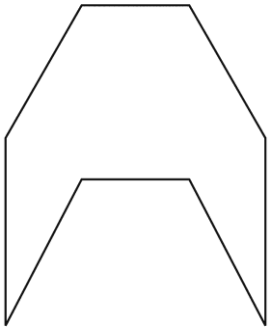
Likewise, a hexagon can be divided into 4 triangles with a total of 720°.



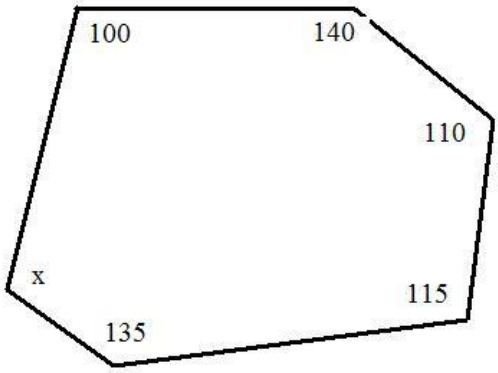
Using this same pattern we can figure out the sum of the measure of the interior angles of any polygon. If n is the number of sides of the polygon, then the sum of the measures of the interior angles is

$(n-2) \times 180°$

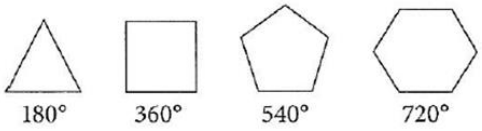
10. What is the sum of the interior angles of this irregular octagon?



11. What is the measure of angle x?



12. The following figures show regular polygons and the sum of the degrees of the angles in each polygon. Based on these figures, what is the number of degrees in an n-sided regular polygon?

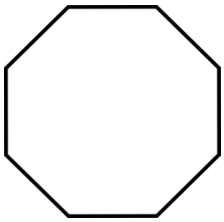


A. $180(n-2)$
B. $180n$
C. $60n$
D. $20n^2$
E. Cannot be determined from the information given.

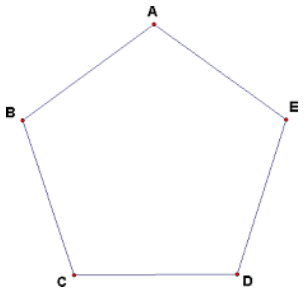
Interior angles of "regular" polygons

Regular polygons are shapes where all of the angles are the same and all of the side lengths are the same. We can determine the sum of the interior angles of any polygon. If the angles are all the same, we can simply divide the total by the number of angles to find the measure of one of the angles. The formula is $\frac{(n-2) \times 180^\circ}{n}$.

13. What is the measure of each angle in this regular octagon?



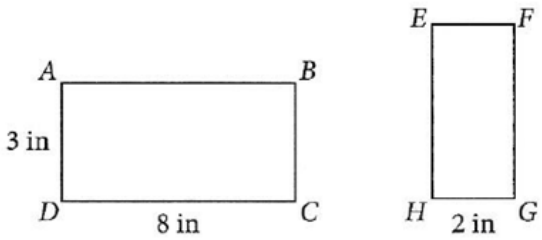
14. What is the measure of angle C in the regular polygon to the right?



Similar polygons

We've already learned about similar triangles and how they are proportional. Similar polygons are also proportional, so you can set up a ratio to solve for missing information. All squares are similar, can you think why that would be?

15. Rectangles ABCD and EFGH shown are similar. Using the given information, what is the length of side FG, to the nearest tenth of an inch?



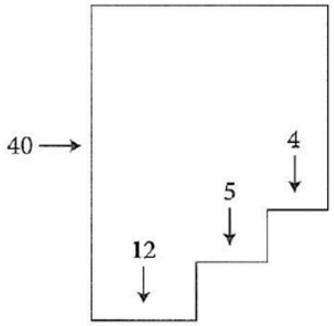
16. The ratio of the sides of two squares is 2: 3. What is the ratio of the perimeters of these squares?

17. Two similar triangles have perimeters in the ratio 3: 4. The sides of the smaller triangle measure 3 in, 4 in, and 5 in. What is the perimeter, in inches, of the larger triangle?

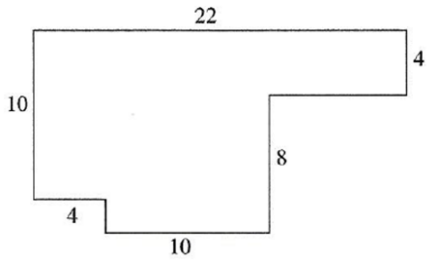
Questions reviewing perimeter and area

18. What is the area, in square inches, of a square with a side length of 6 inches?
19. The rectangular soccer field at the Recreational Park is twice as long as it is wide. The perimeter of the field is 300 yards. What is the width, in yards, of the soccer field?

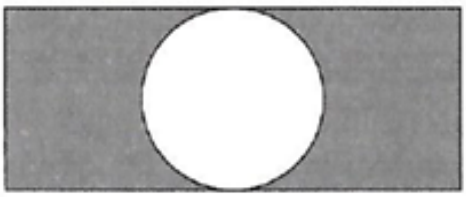
20. In the following figure, all interior angles are 90°, and all dimension lengths are given in inches. What is the perimeter of this figure in inches?



21. All line segments that intersect in the polygon below do so at right angles. If the dimensions given are in inches, that what is the area of the polygon, in square inches?

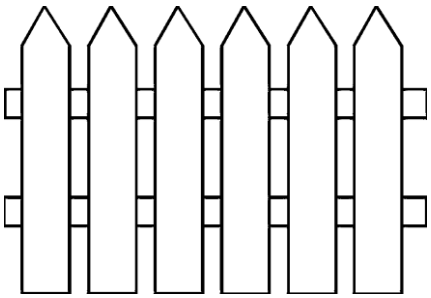


22. In the figure below, the top and bottom of the rectangle are tangent to the circle as shown. The rectangle has a length of $4x$ and width of $2x$. What is the area of the shaded region?

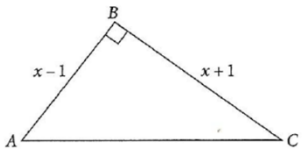


A. $8\pi^2 + 8x$
B. $8x^2 - \pi$
C. $8x - \pi x$
D. $8x^2 - \pi x^2$
E. $8x^2 - 2\pi x$

23. The figure below is a section of a picket fence panel. Each picket is symmetrical along its center vertical axis. Each picket is 6 inches wide. From the base, each piece has a vertical measure of 32 inches before the tapering begins. The height from the base to the highest tip is 36 inches. What is the front surface area of a single picket in square inches?



24. In the following figure, AB is perpendicular to BC. The lengths of AB and BC, in inches, are given in terms of x. Which of the following represents the area of triangle ABC, in square inches, for all $x > 1$?

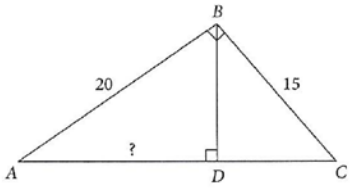


A. $x^2 - 1$
B. x
C. $2x$
D. x^2
E. $\frac{x^2-1}{2}$

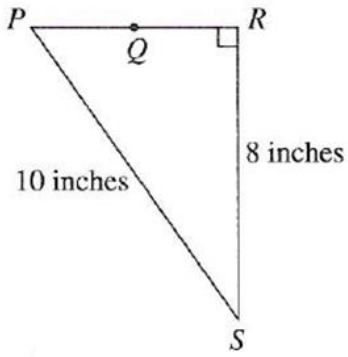
Questions reviewing angles, Pythagorean theorem, and similar triangles

25. How many units long is one of the sides of a square that has a diagonal 16 units in length? (leave answer in terms of square roots)

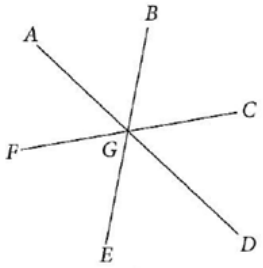
26. In the following figure , AB = 20, BC = 15, and angle ADB and angle ABC are right angles. What is the length of BD?



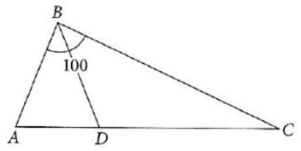
27. In right triangle PRS shown below, Q in the midpoint of PR. What is the length of PQ, to the nearest inch?



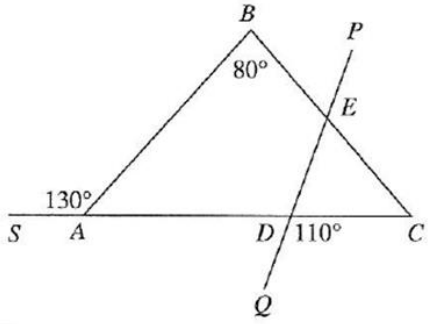
28. In the following figure, AD, BE, and CF all intersect at point G. If the measure of angle AGB is 60° and the measure of angle CGE is 110°, what is the measure of angle AGF?



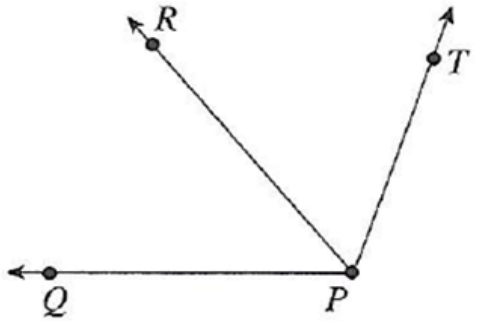
29. In the following figure, BD bisects angle ABC. The measure of angle ABC is 100°, and the measure of BCD is 25°. What is the measure of angle BDA?



30. In the figure below, PQ intersects triangle ABC at points E and D. Angle ABC measures 80°, angle SAB measures 130°, and angle CDQ measures 110°. What is the measure of angle BEP?

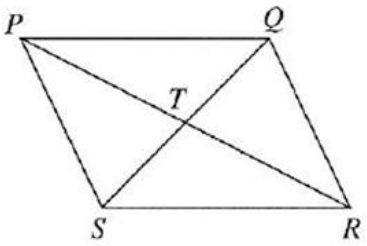


31. In the figure below, the measure of angle QPT is 100°. If the measure of angle RPT is (Y + 17)°, what is the measure of angle QPR?

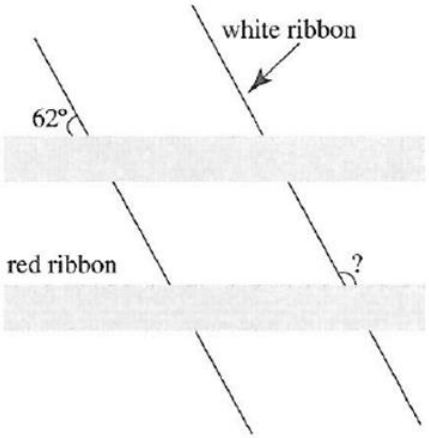


A. (117 - Y)°
B. (117 + Y)°
C. (Y - 83)°
D. (83 - Y)°
E. (83 + Y)°

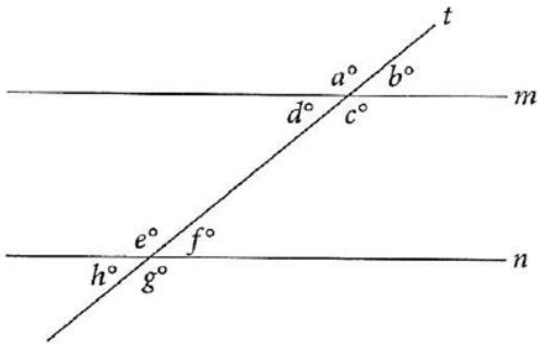
32. In the figure below, PS is parallel to QR, and PR intersects SQ at T. If the measure of angle PST is 60° and the measure of angle QRT is 30°, then what is the measure of angle PTQ?



33. Susan is wrapping a gift for her Uncle Fred. She uses red and white ribbons around blue wrapping paper since Uncle Fred's birthday is July 4th. She ties one strip of white ribbon around the box at an angle and makes a second strip of ribbon parallel to the first. Then she ties two parallel strips of red ribbon which make a 62° angle from the white ribbon as shown below. What is the degree measure of the angle, indicated below, between the white ribbon on the right and the bottom red ribbon?

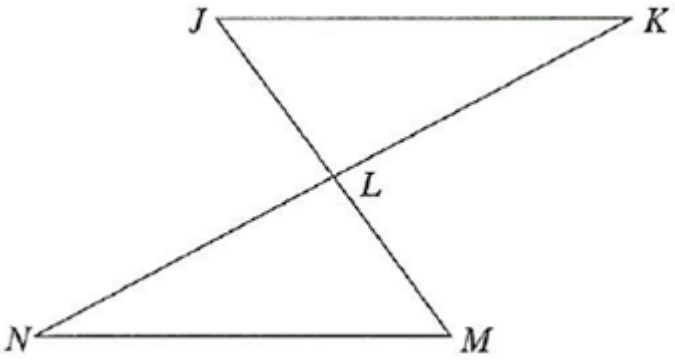


34. In the following figure, line t crosses parallel lines m and n. Which of the following statements must be true?



A. d = c
B. a = d
C. b = e
D. f = g
E. d = h

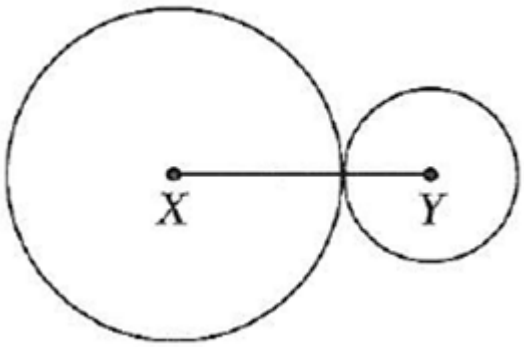
35. In the figure below, JK is parallel to MN, and JM and KN intersect at L. Which of the following statements must be true?



A. Triangle JKL is congruent to triangle MNL
B. JL is congruent to LM
C. JK is congruent to MN
D. JM bisects KN
E. Triangle JKL is similar to MNL

Questions reviewing area and circumference of circles

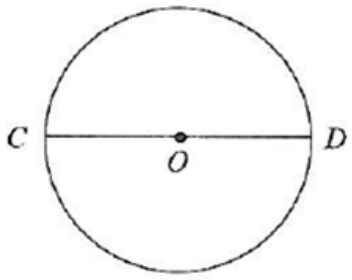
36. The figure below shows two tangent circles. The circumference of circle X is 10π, and the circumference of circle Y is 6π. what is the greatest possible distance between two points, one of which lies on the circumference of circle X and one of which lies on the circumference of circle Y?



37. If a square has an area of 49 square inches, what is the area of the largest circle that can be inscribed inside the square?

38. A costume designer wants to put a while silk band around a cylindrical top hat. If the radius of the cylindrical part of the top hat is 5 inches, how long, in inches, should the white silk band be to just fit around the top hat? (leave answer in terms of π)

39. In the figure below, O is the center of the circle, C and D are points on the circle, and C, O, and D are collinear. If the length of CD is 6 inches, what is the area, in square inches, of the circle? (leave answer in terms of π)



Answers

1. 5
2. 13
3. 15
4. 52 m
5. A
6. B
7. C
8. B
9. D
10. 1080°
11. 120°
12. A
13. 135°
14. 108°
15. 5.3 in
16. 2:3
17. 16 in
18. 36 in^2
19. 50 yards
20. 122 inches
21. 192 in^2
22. D
23. 204 in^2
24. E
25. $8\sqrt{2}$
26. 12
27. 3 in
28. 50°
29. angle BDA 75°
30. 60°
31. D
32. 90°
33. 118°
34. E
35. E
36. 8
37. 12.25π or $38.465in^2$
38. 10π
39. 9π or $28.26in^2$