8th Grade Honors - Proportions & Probability Part 2

PERCENT INCREASE AND DECREASE

To increase a number by a percent, **add the percent to 100%,** convert to a decimal, and multiply. To increase 40 by 25%, add 25% to 100%, convert 125% to 1.25, and multiply by 40. 1.25 x 40 = 50. To decrease, just subtract the percent from 100%, convert to a decimal, and multiply.

Sample Questions:

1. To keep up with rising expenses, a motel manager needs to raise the $40.00 room rate by 22%. What will be the new rate?

2. Jorge's current hourly wage for working at Denti Smiles is $12.00. Jorge was told that at the beginning of next month, his new hourly wage will be an increase of 6% of his current hourly wage. What will be Jorge's new hourly wage?

3. A shirt is priced at $32.00 now. If the shirt goes on sale for 30% off the current price, what will be the sale price of the shirt?

4. To keep up with rising costs, a plumber needs to increase his $40.00 per hour rate by 12%. What will be his new hourly rate?

5. Disregarding sales tax, how much will you save when you buy a $28.00 game that is on sale for 20% off?

FINDING THE ORIGINAL WHOLE

To find the **original whole before a percent increase or decrease,** set up an equation with a variable in place of the original number. Say you have a 15% increase over an unknown original amount, say *x.* You would follow the same steps as always: 100% plus 15% is 115%, which is 1.15 when converted to a decimal. Then multiply to the number, which in this case is *x,* and you get 1.15x, then set that equal to the "new" amount.

Example: After a 5% increase, the population was 59,346. What was the population *before* the increase?
Solution: $1.05x = 59{,}346$
$x = 59{,}346/1.05 = 56{,}520$

Sample Questions:

6. Johnny is trying to figure out his original wage from work. 20% of Johnny's pay check is automatically taken out to pay taxes. The amount of money Johnny receives is $45,500. How much is Johnny's wage BEFORE taxes?

7. Allie ate 30% of all the candy. If the Allie ate 45 pieces, how much candy was there before she started eating it?

8. If 60% of the student body are girls and there are 650 boys in the school, what is the total number of students in the school?

FINDING A PERCENT OF A PERCENT

When you learned how to translate simple English statements into mathematical expressions, you learned that "of" can indicate "times". This frequently comes up when using percentages. To take a percent of a percent of a number, use the same procedure and multiply the percent times the second percent times the number.

Example: Find 15% of 15% of 2000
Solution: Covert percentage to decimal form and multiply: $0.15 \times 0.15 \times 2000 = 45$

Sample Questions:

9. At Paradigm, students must take both a written exam and an oral exam. In the past, 90% of the students passed the written exam and 75% of those who passed the written exam also passed the oral exam. Based on these figures, about how many students in a random group of 350 students would you expect to pass both exams?

10. Amazon reported that 89% of their customers who purchased mp3 players gave the mp3 players a 5-star rating. 53% of the customers that gave the mp3 players 5 star ratings said that the mp3 players were reliable. There were 5,324 customers who purchased mp3 players. How many gave 5-star ratings and specified that the mp3 players were reliable?

11. Only tenth-, eleventh-, and twelfth-grade students attend Washington High School. The ratio of tenth graders to the school's total student population is 86:255, and the ratio of eleventh graders to the school's total student population is 18:51. If 1 student is chosen at random from the entire school, which grade is that student most likely to be in?

12. What is 1/4 of 20% of $50,000?

13. Of the 517 graduating seniors at Brighton High School, approximately 4/5 will be attending college, and approximately 1/2 of those going to college will be attending a state college. Which of the following is the closest estimate of the number of graduating seniors who will be attending state college?

AVERAGE FORMULA

To find the average of a set of numbers, **add them up and divide by the number of numbers.**

Average $= \dfrac{\text{Sum of the terms}}{\text{Number of the terms}}$

Example: Find the average of the five numbers 12, 15, 23, 40, and 40.
Solution: First add them: 12 + 15 + 23 + 40 + 40 = 130. Then divide the sum by 5: 130 ÷ 5 = 26.

The average formula works for more complicated numbers: fractions, percentages, complicated numbers with variables, etc. To solve the more complicated problems, follow sound mathematical practices to get the correct answer.

Example: What is the average of 1/20 and 1/30?
Solution: To get the average, you must add the fractions and divide by two. Don't just average the denominators!!!

Average $= \dfrac{sum}{number\ of\ terms} = \dfrac{\frac{1}{20}+\frac{1}{30}}{2} = \dfrac{\frac{3}{60}+\frac{2}{60}}{2} = \dfrac{\frac{5}{60}}{2} = \dfrac{\frac{1}{12}}{2} = \dfrac{1}{12} \times \dfrac{1}{2} = \dfrac{1}{24}$

If you averaged the denominators you would have gotten 1/25, but the answer is 1/24. Don't be fooled!

Example: What is the average of the expressions 2x + 5, 5x – 6, -4x + 2 ?

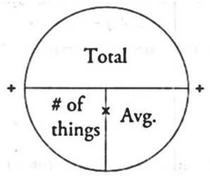
a. x + 1/3
b. x + 1
c. 3x + 1/3
d. 3x + 3
e. 3x + 3 1/3

Solution: Don't let the algebraic terms confuse you, simply add them and divide by 3.

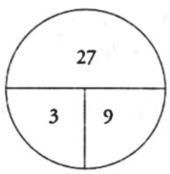
Average $= \dfrac{sum}{number\ of\ terms} = \dfrac{(2x+5)+(5x-6)+(-4x+2)}{3} = \dfrac{(3x+1)}{3} = x + \dfrac{1}{3}$ or answer A

AVERAGE: THE PIE METHOD

There are only three parts to any average question. The question must give you two of these parts, which are all you need to find the third. The average pie is an easy way to keep track of the information you get from questions dealing with averages. If you have the total, you can always divide by either the average or the number of things in the set (whichever you are given) to find the missing piece of the pie. Similarly, if you have the number of things and the average, you can multiply the two together to arrive at the total (the sum of all the items in the set).



For example, if you want to find the average of 9, 12, and 6 using the average pie, you know you have 3 items with a total of 27. Dividing the total, 27, by the number of things, 3, will yield the average, 9. Your pie looks like this



Although you probably could have done that without the average pie, more difficult average questions involve multiple calculations and lend themselves particularly well to using the pie.

Example: Over 9 games, a baseball team had an average of 8 runs per game. If the average number of runs for the first 7 games was 6 runs per game, and the same number of run was scored in each of the last 2 games, how many runs did the team score during the last game?

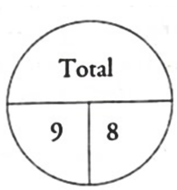
a. 5
b. 15
c. 26
d. 30
e. 46

Solution:

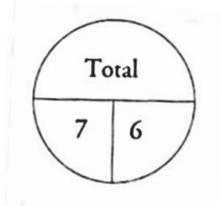
Step 1: **Know the question.** How many runs did the team score during the last game?

Step 2: **Let the answers help.** Eliminate choice (A}. Even though the average for the first 7 is higher than all 9, the runs scored in the last two games can't be that few. Similarly, choice (E) is probably too big. If you don't trust your sense of numbers and you're not comfortable Ballparking here, however, leave both. It's a complicated question on a more advanced topic.

Step 3: **Break the problem into bite-sized pieces.** Use bite-size pieces to put the information from the first line of this problem into our trusty average pie.



What is the sum of everything for these 9 games? 9 x 8, or 72. Now put the information from the second line into the average equation.



What is the sum of everything for these 7 games? 7 x 6, or 42. If all 9 games added up to 72, and 7 of these games added up to 42, then the remaining 2 games added up to 72 -42, or 30. In case you are feeling smug about getting this far, the ACT writers made 30 the answer for choice (D). But of course you know that they only want the runs they scored in the last game. Because the same number of runs was scored in each of the last two games, the answer is 30/2 or 15, choice (B).

Sample Questions:

14. The daily totals of enrollments at Jerry's cooking class last Monday through Saturday were 24, 27, 23, 24, and 27. What was the average number of enrollments per day?

15. What is the average of 3/4 and 4/5?

16. As part of a school report on the cost of milk, Sammy wants to find the average cost of milk from local stores. She surveys 4 stores and finds the cost per gallon of whole milk from the 4 stores to be $2.38, $2.50, $2.79, $2.46, respectively. Using this data, what is the average cost of purchasing one gallon of whole milk from these 4 gas stores?

THE WEIGHTED AVERAGE

ACT writers have a particular fondness for "weighted average" problems. First, let's look at a regular unweighted average question.

Regular Average Example: If Sally received a grade of 90 on a test last week and a grade of 100 on a test this week, what is her average for the two tests? Piece of cake, right? The answer is 95. You added the scores and divided by 2.

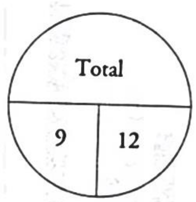
Now let's turn the same question into a weighted average question.

Weighted Average Example: If Sally's average for the entire year last year was 90, and her average for the entire year this year was 100, is her average for the two years combined equal to 95? The answer is "not necessarily." If Sally took the same number of courses in both years, then yes, her average is 95. But what if last year she took 6 courses while this year she took only 2 courses? Can you compare the two years equally? ACT likes to test your answer to this question.

To solve weighted average problems, break it down by using the pie method or use a table to help organize the information and write an equation for the solution as follows.

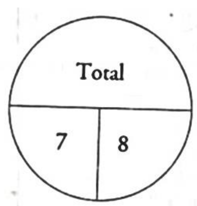
Pie Method Example: The starting team of a baseball club has 9 members who have an average of 12 home runs apiece for the season. The second-string team for the baseball club has 7 members who have an average of 8 home runs apiece for the season. What is the average number of home runs for the starting team and the second-string team combined?

Pie Method Solution: The ACT test writers want to see whether you spot this as a weighted average problem. If you thought the first-string team was exactly equivalent to the second-string team, then you merely had to take the average of the two averages, 12 and 8, to get 10. In weighted average problems, the ACT test writers always include the average of the two averages among the answer choices, and it is always wrong. 10 is choice (C). Cross off choice (C).
The two teams are not equivalent because there are different numbers of players on each team. To get the true average, we'll have to find the total number of home runs and divide by the total number of players. How do we do this? By going to the trusty average formula as usual. The first line of the problem says that the 9 members on the first team have an average of 12 runs apiece.



So the sum of everything is 9 x 12, or 108.

The second sentence says that the 7 members of the second team have an average of 8 runs each.



So the total is 7 x 8, or 56.

Now we can find the true average. Add all the runs scored by the first team to all the runs scored by the second team: 108 + 56 = 164. This is the true total. We divide it by the total number of players (9 + 7 = 16). The answer is 10.25, or choice (D).

Table/Equation Example: An online seed supplier packages a seed mix that costs the company $30.30 per pound. The mix includes poppy seeds costing $35.65 per pound and clover seeds costing $8.90 per pound. If a worker is going to prepare some of this mix and has already measured out 15 pounds of poppy seeds, what quantity of clover seeds should he add? *Write your answer as a whole number or as a decimal rounded to the nearest tenth.*

Table/Equation Solution: Step 1: Define a variable and make a table. Let *x* represent the quantity of poppy seeds. Now make a table. Start by filling in the first two columns with what you know.

	Price per pound	Number of pounds	Total price
Poppy seeds	35.65	15	
Clover seeds	8.9	x	
Seed mix	30.3		

Next, finish filling in the second column. Since there are 15 pounds of poppy seeds and *x* pounds of clover seeds, there are a total of 15 + *x* pounds of seed mix. Fill in 15 + *x*.

	Price per pound	Number of pounds	Total price
Poppy seeds	35.65	15	
Clover seeds	8.9	x	
Seed mix	30.3	$15 + x$	

Finally, multiply across to fill in the third column.

	Price per pound	Number of pounds	Total price
Poppy seeds	35.65	15	35.65(15)
Clover seeds	8.9	x	$8.9x$
Seed mix	30.3	$15 + x$	$30.3(15 + x)$

Step 2: Write an equation and solve. Using the information in the table, write an equation.
Total price of poppy seeds plus total price of clover seeds equals total price of seed mix.
$35.65\ (15) + 8.9\text{x} = 30.3(15 + \text{x})$
Solve the equation
$35.65(15) + 8.9x = 30.3(15 + x)$
$534.75 + 8.9x = 454.5 + 30.3x$
$534.75 – 21.4x = 454.5$
$-21.4 = -80.25$
$X \approx 3.8$
The worker should add about 3.8 pounds of clover seeds.

Example: In a certain club, the average age of the male members is 35, and the average of the female members is 25. If 20% of the members are male, what is the average age of all the club members?

a. 26
b. 27
c. 28
d. 29
e. 30

Solution: The overall average is not simply the average of the two average ages. Because there is a lot more women than men, women carry more weight, and the overall average will be closer to 25 than 35. Pick particular numbers for the females and males, say 4 females and 1male. The ages of the 4 females total 4 times 25, or 100, and the age of the 1 male totals 35. The average then, is (100 + 35) divided by 5, or 27 or Answer B.

Sample Questions:

17. Pamela makes sachets using dried flowers and herbs. Her special blend costs $10.70 per kilogram, combining dried lavender at $14.15 per kilogram and dried chamomile at $7.25 per kilogram. Pamela is preparing a new batch of the blend and has measured out 2 kilograms of lavender. How much chamomile should she add to complete her sachets?

18. Duncan works for a company that manufactures liquid dyes for clothing. He currently wants to mix up some 50%-concentrated white dye. He has 24 gallons of 56%-concentrated white dye, as well as plenty of 36%-concentrated white dye. How many gallons of the 36%-concentrated white dye will Duncan need to add to the 56%-concentrated white dye to make a batch with a concentration of 50%?

19. For a college chemistry experiment, students need to prepare a solution containing ethylene glycol. They are each given 16 fluid ounces of a solution containing 13% ethylene glycol, to which they will add another solution that contains 19% ethylene glycol. How much of the 19% solution should they add to obtain a 16% ethylene glycol solution?

20. A factory is using eugenol, a compound extracted from cinnamon and cloves, in some of its products. The factory has just 15 gallons of oil that contains 84% eugenol, in addition to plenty of oil containing 59% eugenol. How many gallons of the oil with 59% eugenol should be added to the oil with 84% to obtain a product with 74% eugenol%?

21. A candy maker is mixing up some sugar syrup for use in making sweets. How much of 49% sugar syrup should she add to 4 liters of 9% sugar syrup to obtain a mixture that contains 45% sugar?

22. A landscaper wants to use a mixture of soil and sand to ensure proper water drainage. He has just 47 cubic meters of soil that is 32% sand and an unlimited quantity of soil that is 38% sand. How many cubic meters of the 38% soil should he add to the 32% to create a mixture that is 35% sand?

MEAN, MEDIAN, MODE, RANGE

Mean

The "mean" is the "average", where you add up all the numbers and then divide by the number of numbers.

Median

The "median" is the "middle" value in the list of numbers, like the median strip on the highway. To find the median, your numbers have to be listed in numerical order, so you may have to rewrite your list first.

Mode

The "mode" is the value that occurs most often--mOde, mOst. If no number is repeated, then there is no mode for the list.

Range

The "range" is just the difference between the largest and smallest values.

Sample Questions:

23. What is the difference between the mean and the median of the set {2, 5, 18, 22} ?

24. To increase the mean of 4 numbers by 2, by how much would the sum of the 4 numbers have to increase?

25. The weekly salaries of six employees at McDonalds are $140, $220, $90, $180, $140, $200. For these six salaries, find: (a) the mean (b) the median (c) the mode and (d) the range.

26. A storeowner kept a tally of the sizes of suits purchased in her store. Which measure of central tendency should the storeowner use to describe the average size suit sold?

27. A tally was made of the number of times each color of crayon was used by a kindergarten class. Which measure of central tendency should the teacher use to determine which color is the favorite color of her class?

28. The science test grades are posted. The class did very well. All students taking the test scored over 75. Unfortunately, 4 students were absent for the test and the computer listed their scores as 0 until the test is taken. Assuming that no score repeated more times than the 0's, what measure of central tendency would most likely give the the best representation of this data?

Use the following information to answer the next three problems:
In January of 2006, your family moved to a tropical climate. For the year that followed, you recorded the number of rainy days that occurred each month. Your data contained 14, 14, 10, 12, 11, 13, 11, 11, 14, 10, 13, 8.

29. Find the mean, mode, median and range for your data set of rainy days.

30. If the number of rainy days doubles each month in the year 2007, what will be the mean, mode, median and range for the 2007 data?

31. If, instead, there are three more rainy days per month in the year 2007, what will be the mean, mode, median and range for the 2007 data?

USING THE AVERAGE TO FIND THE SUM

Since the average is just the sum of the numbers divided by the number of numbers, it follows that:

Sum= (Average) x (Number of terms}

If the average of 10 numbers is 50, then they add up to 10 x 50, or 500

Sample Questions:
32. The average of 5 numbers is 77, what is the total of all five numbers?

33. A group of 25 numbers add up to 500. What is the average of these numbers?

FINDING THE MISSING NUMBER

To find a missing number when you're given the average, **use the sum.** If the average of four numbers is 7, then the sum of those four numbers is 4 x 7, or 28. Suppose that three of the numbers are 3, 5, and 8. These numbers add up to 16 of that 28, which leaves 12 for the fourth number.

Example: Martin's average score after four tests is 89. What score on the fifth test would bring Martin's average up to exactly 90?

Solution: The best way to deal with changing averages is to go by the way of the sums. Use the old average to figure out the total of the first 4 scores:

Sum of first 4 scores = 4 x 89 = 356 And use the new average to figure out the total he needs after the fifth score.

Sum of 5 scores = 4 x 90 = 450 To get his sum up from 456 to 450, Martin needs to score 450 – 356 = 94

Sample Questions:

34. Tom has taken 5 of the 8 equally weighted tests in his U.S. History class this semester, and he has an average score of exactly 78.0 points. How many points does he need to earn on the 6th test to bring his average score up to exactly 80.0 points?

35. Marlon is bowling in a tournament and has the highest average after 5 games, with scores of 210, 225, 254, 231, and 280. In order to maintain this exact average, what *must* be Marlon's score for his 6th game?

36. Chris is on a mountain bike team that is earning money to attend the state competition. There are 15 members on the team and they have earned $525 so far. If the original cost per team member for the trip is $75, how much more does each team member need to earn in order to pay for the trip?

37. In Kara's math class, all tests count equally. So far, Kara has taken 4 of the 5 tests in math class this marking period and earned scores of 88%, 95%, 86% and 79% respectively. What is the minimum score Kara needs on the third test to have a test average of 85%?

38. Andy has grades of 84, 65, and 76 on three math tests. What grade must he obtain on the next test to have an average of exactly 80 for the four tests?

39. Linda computed the average of her six biology test scores by mistakenly adding the totals of five scores and dividing by five, giving her an average score of 88. When Linda realized her error, she recalculated and included the sixth test score of 82. What is the average of Linda's six biology tests?

AVERAGE OF EVENLY SPACED NUMBERS

When you have a set of evenly spaced numbers, such as 10 consecutive numbers, or all the odd numbers between 5 and 35, you can use a shortcut rather than the averages formula. To find the average of evenly spaced numbers, just **average the smallest and the largest.** The average of all the integers from 13 through 77 is the same as the average of 13 and 77.

$$\frac{13+77}{2} = \frac{90}{2} = 45.$$

Sample Questions:

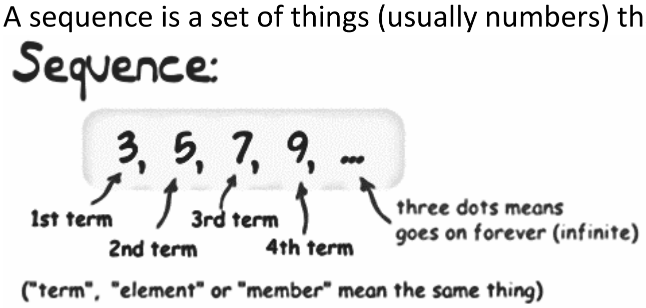
40. What is the average of all integers from 15 through 99?

41. What is the difference between the average of all consecutive numbers from 33 through 99 and the average of all even numbers from 2 through 24?

42. What is the average of the set {3, 5, 7, 9, 11, 13, 15, 17, 19, 21}?

SEQUENCES AND PATTERNS

A sequence is a set of things (usually numbers) that are in order.



ARITHMETIC SEQUENCES

In an Arithmetic Sequence the difference between one term and the next is a constant. In other words, we just add the same value each time ... infinitely.

For illustration purposes, look at the sequence: 1, 4, 7, 10, 13, 16, 19, 22, 25, ...

This sequence has a difference of 3 between each number. In general, we could write an arithmetic sequence like this:

{a, a+d, a+2d, a+3d, ... }

where: a is the first term, and d is the difference between the terms (called the "common difference")

In the example: a = 1 (the first term) and d = 3 (the "common difference" between terms)

And we get:
{a, a+d, a+2d, a+3d, ... }
{1, 1+3, 1+2×3, 1+3×3, ... }
{1, 4, 7, 10, ... }

RULE FOR ARITHMETIC SEQUENCES
We can write an Arithmetic Sequence as a rule:
$x_n = a + d(n-1)$
(We use "n-1" because d is not used in the 1st term).
Example: Write the Rule, and calculate the 4th term for
3, 8, 13, 18, 23, 28, 33, 38, ...
Solution: This sequence has a difference of 5 between each number.
The values of a and d are: a = 3 (the first term) and d = 5 (the "common difference")
The Rule can be calculated:
$x_n = a + d(n-1)$
$= 3 + 5(n-1)$
$= 3 + 5n - 5$
$= 5n - 2$
So, the 4th term is:
$x_4 = 5×4 - 2 = 18$
Arithmetic Sequences are sometimes called Arithmetic Progressions (A.P.'s)

Sample Question:
43. What is the fourth term in the arithmetic sequence 13, 10, 7, ...?

44. Which of the following statements is NOT true about the arithmetic sequence 16, 11, 6, 1. . .?
   a. The fifth term is -4.
   b. The sum of the first 5 terms is 30.
   c. The seventh term is -12
   d. The common difference of consecutive integers is -5.
   e. The sum of the first 7 terms is 7.

45. The greatest integer of a set of consecutive even integers is 12. If the sum of these integers is 40, how many integers are in this set?

GEOMETRIC SEQUENCE

In a Geometric Sequence each term is found by multiplying the previous term by a constant.
For illustration, look at the sequence: 2, 4, 8, 16, 32, 64, 128, 256, . . .
This sequence has a factor of 2 between each number.
Each term (except the first term) is found by multiplying the previous term by 2.

In general, we write a Geometric Sequence like this:
{a, ar, ar2, ar3, ... }
where: a is the first term, and r is the factor between the terms (called the "common ratio")

Example: {1,2,4,8,...}
Solution: The sequence starts at 1 and doubles each time, so a=1 (the first term) and r=2 (the "common ratio" between terms is a doubling)
And we get:
{a, ar, ar2, ar3, ... }
= {1, 1×2, 1×22, 1×23, ... }
= {1, 2, 4, 8, ... }
But be careful, r should not be 0:
When r=0, we get the sequence {a,0,0,...} which is not geometric

RULE FOR GEOMETRIC SEQUENCES

We can also calculate any term using the Rule:
x_n = ar(n-1)
(We use "n-1" because ar0 is for the 1st term)

Example: 10, 30, 90, 270, 810, 2430, ...
Solution: This sequence has a factor of 3 between each number.
The values of a and r are: a = 10 (the first term) and r = 3 (the "common ratio")
The Rule for any term is:
x_n = 10 × 3(n-1)
So, the 4th term is:
x_4 = 10×3(4-1) = 10×33 = 10×27 = 270
And the 10th term is:
x_{10} = 10×3(10-1) = 10×39 = 10×19683 = 196830

A Geometric Sequence can also have smaller and smaller values:
Example: 4, 2, 1, 0.5, 0.25, ...
Solution: This sequence has a factor of 0.5 (a half) between each number.
Its Rule is x_n = 4 × (0.5)n-1

Geometric Sequences are sometimes called Geometric Progressions (G.P.'s)

Example Questions:
46. What is the next term after -1/3 in the geometric sequence 9, -3, 1, -1/3, . . . ?

47. Which of the following statements is NOT true about the geometric sequence 26, 18, 9, . . . ?
    a. The fourth term is 4.5
    b. The sum of the first five terms is 69.75
    c. Each consecutive term is ½ of the previous term
    d. Each consecutive term is evenly divisible by 3
    e. The common ratio of consecutive terms is 2:1

48. The first term is 1 in the geometric sequence 1, -3, 9, -27, ... What is the SEVENTH term in the geometric sequence?

SOLVING PROBLEMS USING THE SLOT METHOD

Combination problems ask you how many different ways a number of things could be chosen or combined. The rules for combination problems on the ACT are straightforward.

- Figure out the number of slots you need to fill.
- Fill in those slots.
- Find the product.

On a more difficult problem, you may run into a combination with more restricted elements. Just be sure to read the problem carefully before attempting it. If the question makes your head spin, leave it and return to it later, or pick your Letter of the Day and move on. For good measure, though, here's what one of those tougher ones might look like.

Example: At the school cafeteria, 2 boys and 4 girls are forming a lunch line. If the boys must stand in the first and last places in line, how many different lines can be formed?

Solution: These restrictions might make this problem seem daunting, but this is where the slot method is really helpful. We have six spots in line to fill, so draw six slots:

___ ___ ___ ___ ___ ___

Fill in the restricted spots first. The problem tells us that the two boys must stand in the first and last places in line. This means that either of the boys could stand in first place, and then the other boy will stand in last. This means that we have two options for the first place, but only one for the last.

_2_ ___ ___ ___ ___ _1_

Now do the same with the unrestricted parts. Any of the four girls could stand in the second spot.

_2_ _4_ ___ ___ ___ _1_

Now, since one of the girls is standing in the second spot, there are only three left to stand in the third spot, and so on, and so on.

_2_ _4_ _3_ _2_ _1_ _1_

Now, as ever, just take the product, co find that the correct answer is choice (C), 48.

_2_ x _4_ x _3_ x _2_ x _1_ x _1_ = 48

Sample Questions:

49. In the word HAWKS, how many ways is it possible to rearrange the letters if none repeat and the letter W must go last?

50. A classroom has 10 tables that will seat up to 4 students each. If 20 students are seated at tables, and NO tables are empty, what is the greatest possible number of tables that could be filled with students?

    1. How many different positive three-digit integers can be formed if the three digits 3, 4, and 5 must be used in each of the integers?

51. The greatest integer of a set of consecutive even integers is 12. If the sum of these integers is 40, how many integers are in this set?

COUNTING CONSECUTIVE INTEGERS

To find the number of consecutive integers between two values, **subtract the smallest from the largest and add 1.** So to find the number of consecutive integers from 13 through 31, subtract: $31 - 13 = 18$. Then add 1: $18 + 1 = 19$. There are 19 consecutive integers from 13 through 31.

USING THE SLOT METHOD TO FIND CONSECUTIVE INTEGERS

First write the sequence including the unknown numbers by leaving slots to fill in the numbers. Then find the "common difference" between the first and last terms in the sequence. Divide the difference by the number of "jumps" between the first and last integer.

Example: What two numbers should be placed in the blanks below so that the difference between successive entries is the same?

26, _____, _____, 53

a. 36, 43
b. 35, 44
c. 34, 45
d. 33, 46
e. 30, 49

Solution: Common Difference = 53-26 = 27. There are three jumps (26 to 2nd term, 2nd term to 3rd term, 3rd term to 53) so divide 27 by 3 = 9. The second term would be 26 + 9 = 35. The third term would be 35 + 9 = 44 (B).

USING ARITHMETIC SEQUENCE TO FIND CONSECUTIVE INTEGERS

Example: What two numbers should be placed in the blanks below so that the difference between successive entries is the same?

26, _____, _____, 53

Solution: In the arithmetic sequence, you can think of the terms as 26, 26 + s, 26 + s + s, and 26 + s + s + s. In the example, s represents the difference between successive terms. The final term is 53, so set up an algebraic equation: 26 + s + s + s = 53. Solve the equation for s, by first combining like terms: 26 + 3s = 53. Subtract 26 from both sides to get 3s = 27. Divide both sides by 3 to find that s, the difference between terms is 9. Therefore, the terms are 26, 26 + 9, 26 + 9 + 9, and 53 or 26, 35, 44, and 53 (B)

Sample Questions:

52. For the difference between consecutive numbers to be the same, which 3 numbers must be placed in the blanks below?

11, _____, _____, _____, 47

53. What two numbers should be placed in the blanks below so that the difference between the consecutive numbers is the same?

13, ___, ___, 34

Answer Key

1. $48.80
2. $12.72
3. $22.40
4. $44.80
5. $5.60
6. 56,875
7. 150 pieces
8. 1,625
9. 236
10. 2,511
11. Twelfth
12. $750
13. 200
14. 18
15. 31/20
16. $2.53
17. Pamela should add 2 kilograms of chamomile to complete the sachets.
18. Duncan should add about 10.3 gallons of the 36%-concentrated white dye.
19. To obtain a 16% ethylene glycol solution, the students should add 16 fluid ounces of the 19% solution.
20. To obtain the desired product, 10 gallons of the oil containing 59% eugenol should be added.
21. To obtain a mixture that contains 45% sugar, 36 liters of the 49% sugar syrup should be added.
22. The landscaper should add 47 cubic meters of the soil that is 38% sand.
23. 6
24. 8
25. Mean: $161.\overline{66}$
    Median: 160
    Mode: 140
    Range 65
26. Mean
27. Mode
28. Median
29. Mean = 11.75
    Mode = 11, 14
    Median = 11.5
    Range = 6
30. Mean = 23.25
    Mode = 22, 28
    Median = 23
    Range – 28 – 16 = 12
31. Mean = 14.75
    Mode = 14, 17
    Median = 14.5
    Range = 6
32. 385
33. 20
34. 90
35. 210
36. $40.00
37. 77%
38. 95
39. 87
40. 57
41. 53
42. 12
43. 4
44. C
45. 5 numbers
46. 1/9
47. D
48. 729
49. 24
50. 3
51. 6
52. 20, 29, 38
53. 20, 27