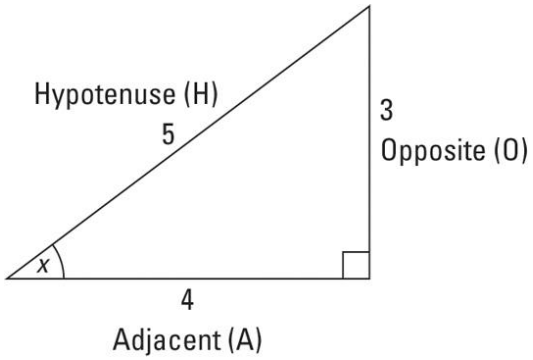
Math 3 Trigonometry Part 1
Review, Reciprocals & Radians

MATH 2 REVIEW:
RATIOS OF SIDES OF A RIGHT TRIANGLE

Trigonometry is all about the relationships of sides of right triangles. In order to organize these relationships, each side is named in relation to an angle. Starting with angle x, there is a side that is adjacent to that angle and there is a side that is opposite of that angle. There is also a hypotenuse. These names are often abbreviated by just the first letter A, O, and H.

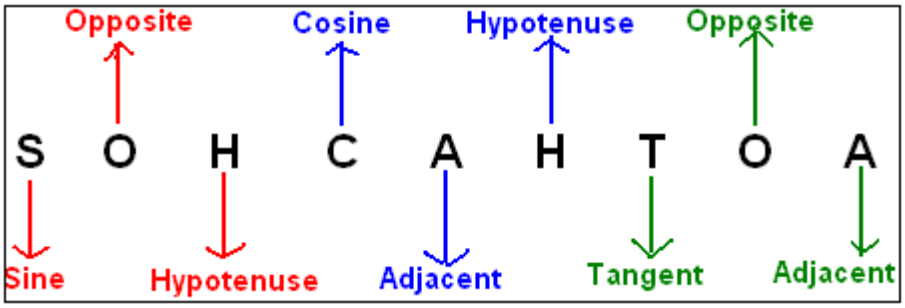


With this angle and these three sides there are six possible relationships. Three of them are the most commonly used. They are called sine, cosine and tangent. These are often abbreviated as sin, cos and tan. (Just for the record, we're only referring to acute angles right now [angles less than 90°], we'll deal with obtuse angles later).

Sine = $\frac{opposite}{hypotenuse}$

Cosine = $\frac{adjacent}{hypotenuse}$

Tangent = $\frac{opposite}{adjacent}$

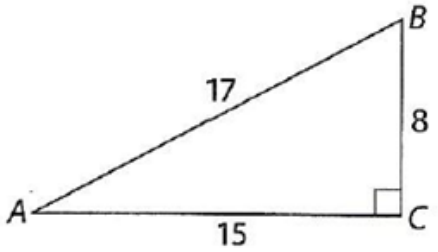


People often use the acronym SOHCAHTOA to help remember which is which. In the triangle below:

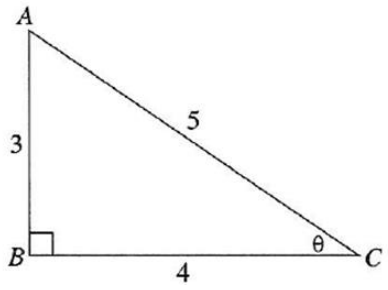
Sine A = $\frac{opposite}{hypotenuse} = \frac{8}{17}$ Sine B = $\frac{opposite}{hypotenuse} = \frac{15}{17}$

Cosine A = $\frac{adjacent}{hypotenuse} = \frac{15}{17}$ Cosine B = $\frac{adjacent}{hypotenuse} = \frac{8}{17}$

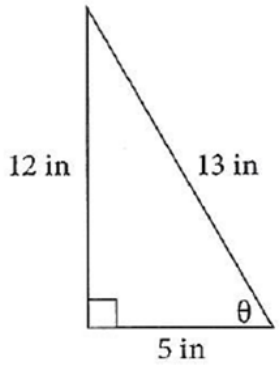
Tangent A = $\frac{opposite}{adjacent} = \frac{8}{15}$ Tangent B = $\frac{opposite}{adjacent} = \frac{15}{8}$



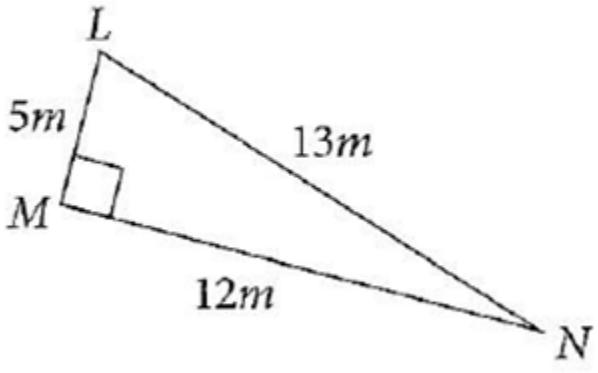
1. In the following figure, angle B is a right angle, and the measure of angle C is θ. What is the value of tan θ?



2. In the right triangle below, cos θ = ?



3. Given the following right triangle, ΔLMN, what is the value of sin N?



4. For angle D in ΔDEF below, which of the following trigonometric expressions has value $\frac{3}{4}$?

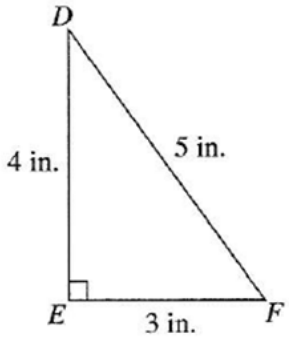
A. sin D

B. tan D

C. cos D

D. sec D

E. csc D



5. To determine the height h of a tree, Roger stands b feet from the base of the tree and measures the angle of elevation to be θ, as shown in the following figure. Which of the following relates h and b? (Hint: the only option given is sin θ, sin = $\frac{oposite}{hypotenuse}$).

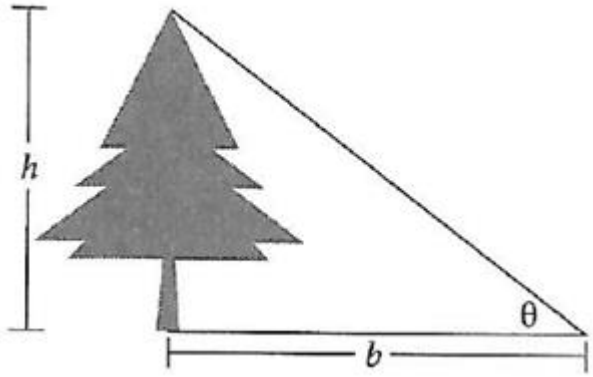
A. sin $\theta = \frac{h}{b}$

B. sin $\theta = \frac{b}{h}$

C. sin $\theta = \frac{b}{\sqrt{b^2+h^2}}$

D. sin $\theta = \frac{h}{\sqrt{b^2+h^2}}$

E. sin $\theta = \frac{\sqrt{b^2+h^2}}{b}$



6. Which of the following trigonometric equations is valid for the side measurement x inches, diagonal measurement y inches, and angle measurement w° in the rectangle shown below?

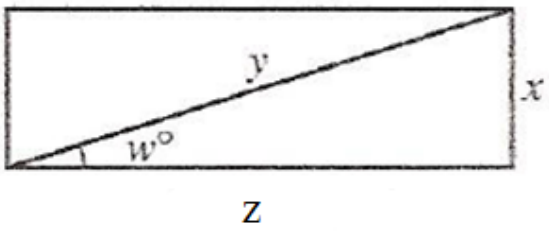
A. cos w° = $\frac{z}{y}$

B. cot w° = $\frac{x}{y}$

C. sec w° = $\frac{x}{y}$

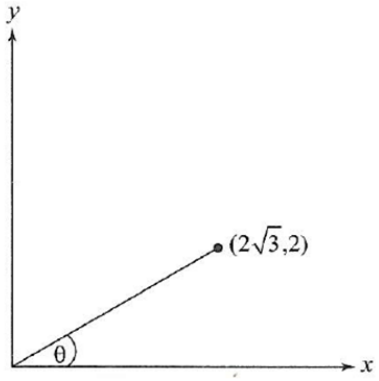
D. sin w° = $\frac{z}{y}$

E. tan w° = $\frac{x}{y}$

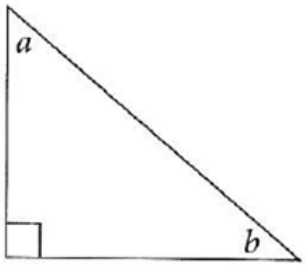


7. In the figure given at right, which of the following trigonometric equations is valid? (Hint: This isn't a triangle... yet. Finish it and label it.)

A. $\tan\theta = \frac{2}{2\sqrt{3}}$
B. $\cot\theta = \frac{2}{2\sqrt{3}}$
C. $\sec\theta = \frac{2}{2\sqrt{3}}$
D. $\sin\theta = \frac{2}{2\sqrt{3}}$
E. $\cos\theta = \frac{2}{2\sqrt{3}}$

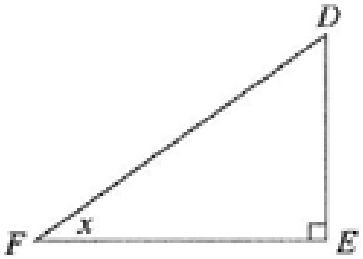


8. In the following figure, $\tan a = \frac{4}{3.}$ What is sin a?

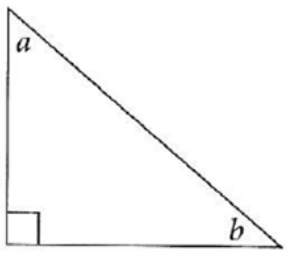


9. In ΔDEF below, DE = 1 and DF = $\sqrt{2}$. What is the value of tan x?

A. $\frac{\sqrt{2}}{2}$
B. 1
C. $\sqrt{2}$
D. $\sqrt{3}$
E. 2

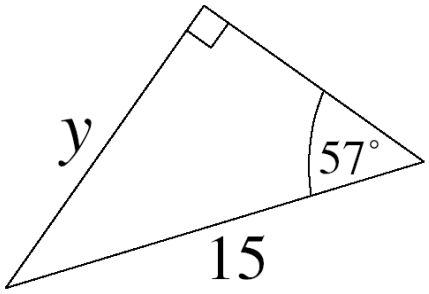


10. In the following figure, $\tan a = \frac{6}{8.}$ What is sin a?



FINDING MISSING NUMBERS WHEN ONE SIDE AND ONE ANGLE ARE KNOWN

In the figure below, one side is given and the measure of one angle is given. I need to solve for the unknown value y. I notice that y is the side opposite of the given angle and I notice that 15 is the hypotenuse. The relationship between the opposite side and the hypotenuse is sine.



Sine = $\frac{opposite}{hypotenuse}$.

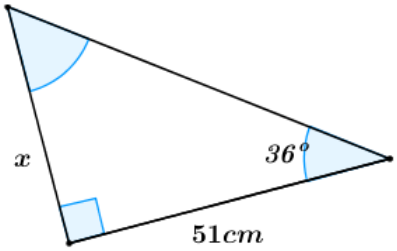
Sine = $\frac{opposite}{hypotenuse} = \frac{y}{15}$. Plug in the values for the triangle.

sin 57° = $\frac{y}{15}$. The angle is 57° so my equation is complete.

15sin 57° = y. Multiply both sides by 15 to solve for y.

In many cases, we stop here and simply say y = 15sin 57°. If we want to continue then we use a calculator. Make sure it is in degree mode. Enter 57 then press the SIN button, you should get 0.838670568. That is the sine value for any triangle with an angle of 57°. Multiply that by 15 to get approximately 12.58. The length of side y is 12.58.

11. In the triangle below, what is the value of x?

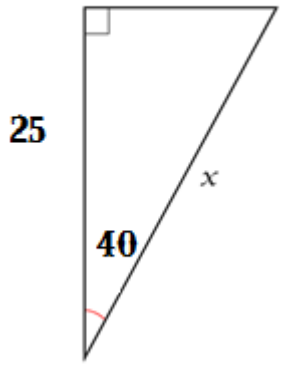


A. x = 51sin36°
B. x = 51cos36°
C. x = 51tan36°
D. x = $\frac{36}{51}$
E. x = $\frac{51}{36}$

12. Using a calculator, what is the approximate value of x (from question 11) rounded to the nearest whole number?

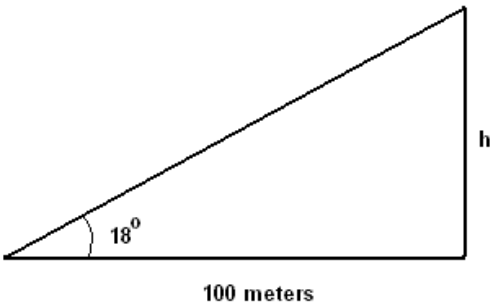
13. In the triangle below, the angle is 40°, what is the value of x?

A. $x = \frac{25}{40}$
B. $x = \frac{25}{sin40°}$
C. $x = \frac{25}{cos40°}$
D. x = 25sin40°
E. x = 25cos40°



14. In the triangle below, what is the value of h?

A. $h = \frac{18}{tan100}$
B. $h = \frac{100}{tan18°}$
C. h = 100tan18°
D. $h = \frac{18}{100}$
E. h = 100sin18°

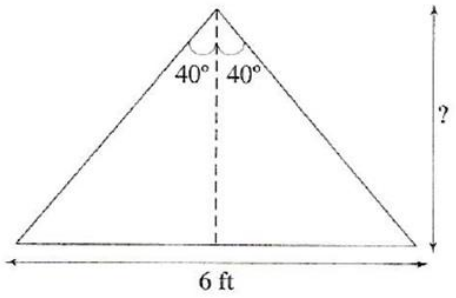


15. A nylon cord is stretched from the top of a playground pole to the ground. The cord is 30 feet long and makes a 20° angle with the ground. Which of the following expressions gives the horizontal distance, in feet, between the pole and the point where the cord touches the ground? (Hint: if the question doesn't give you a diagram, draw one)

A. $\frac{\sin 20°}{30}$
B. $\frac{\cos 20°}{30}$
C. 30 tan 20°
D. 30 sin 20°
E. 30 cos 20°

16. The cross-sectional view of a tent is shown below. If the tent is 6 feet wide at its base, then which of the following expressions could be used to calculate the height of the tent, in feet?

A. $\frac{3}{tan80°}$
B. 3 tan 40°
C. $\frac{3}{tan40°}$
D. 6 $tan40°$
E. 3 $tan80°$



MATH 3 LEVEL

COSECANT, SECANT, COTANGENT

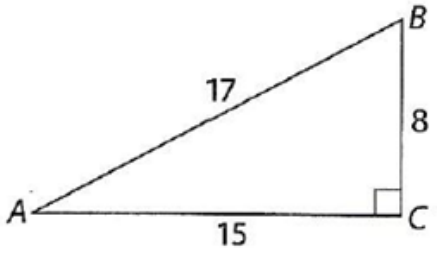
Sine, cosine and tangent aren't the only possible relationships of a right triangle. The other three relationships are cosecant, secant, and cotangent. These are the reciprocals of sine, cosine and tangent. Cosecant is abbreviated as csc, secant is abbreviated as sec, and cotangent is abbreviated as cot. (Just for the record, we're only referring to acute angles right now [angles less than 90°], we'll deal with obtuse angles later).

Cosecant or csc $= \frac{1}{sine} = \frac{hypotenuse}{opposite}$

Secant or sec $= \frac{1}{cosine} = \frac{hypotenuse}{adjacent}$

Cotangent or cot $= \frac{1}{tangent} = \frac{adjacent}{opposite}$

In the figure at the right



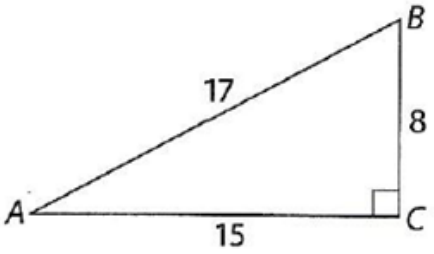
Cosecant A or csc A $= \frac{1}{sine} = \frac{hypotenuse}{opposite} = \frac{17}{8}$

Secant A or sec A $= \frac{1}{cosine} = \frac{hypotenuse}{adjacent} = \frac{17}{15}$

Cotangent A or cot A $= \frac{1}{tangent} = \frac{adjacent}{opposite} = \frac{15}{8}$

Notice that we have to refer to the angle, in this case angle A. We need to know which angle we're starting from in order to know which side is opposite and which is adjacent.

If we are using angle B as the reference point, then we get different answers.



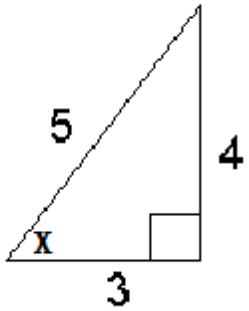
Cosecant B or csc B $= \frac{1}{sine} = \frac{hypotenuse}{opposite} = \frac{17}{15}$

Secant B or sec B $= \frac{1}{cosine} = \frac{hypotenuse}{adjacent} = \frac{17}{8}$

Cotangent B or cot B $= \frac{1}{tangent} = \frac{adjacent}{opposite} = \frac{8}{15}$

Sample Questions:

Refer to the 3-4-5 triangle at the right for the next 3 questions.

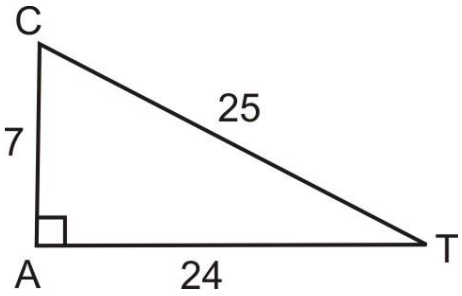


17. What is scs X?

18. What is sec X?

19. What is cot X?

20. In triangle CAT, what is sine T?



21. In triangle CAT, what is cosine T?

22. In triangle CAT, what is tangent T?

23. In triangle CAT, what is cosecant C?

24. In triangle

25. CAT, what is secant C?

26. In triangle CAT, what is cotangent C?

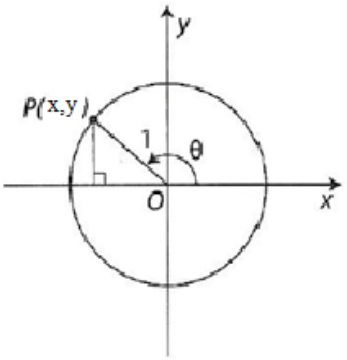
ANGLES GREATER THAN 90°

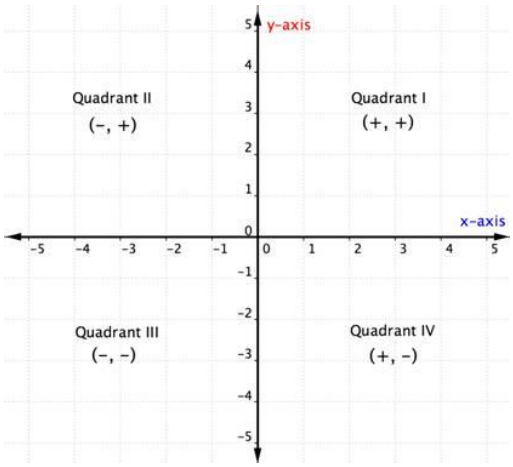
We also need to be able to deal with angles that are greater than 90°. A good way to do that is to draw a diagram. Sketch a circle of radius 1 centered at the origin of the coordinate grid. Start from the point (1,0) and rotate counterclockwise for the angle. In the unit circle setup shown, the basic trigonometric functions are defined in terms of the coordinates x and y. (Basically we're using the little triangle which is less than 90° just as we did before, but now we have to worry about positive and negative values.)

sin θ = y

cos θ = x

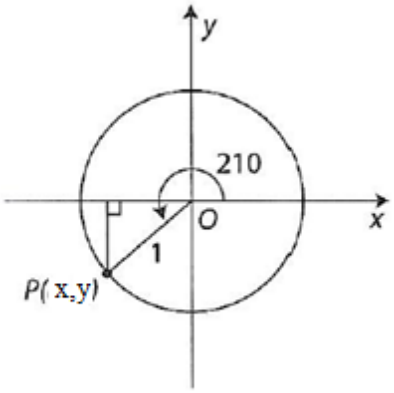
tan $\theta = \frac{y}{x}$





If a point is in quadrant II then the "x" value (cosine) will be negative, which means that the tangent value will also be negative.

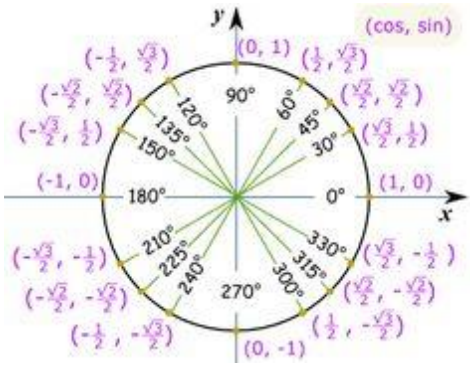
The "y" value is still positive. The tangent will also be negative.



If the point is in quadrant III then both the "x" and "y" values will be negative. This means that the sine and cosine will both be negative, but the tangent value will be positive because a negative divided by a negative is a positive.

Tangent = $\frac{-y}{-x} = \frac{y}{x}$

If the point is in quadrant IV, then the "x" value will be positive and the "y" value will be negative. This means that the cosine will be positive and the sine will be negative and the tangent will be negative.



Don't stress over this too much. The main point is that the pattern of the sine, cosine, and tangent values from $0 \leq x \leq 90°$, which we already know how to do, repeat for each quadrant, however we need to pay attention to positive and negative values. At the figure at the right, let's look at 30°. The y value at that point is $\frac{1}{2}$ so the sin 30° = $\frac{1}{2}$. At 150° (which is the same angle, but reflected across the y axis) the y value of the point is also $\frac{1}{2}$ so sin 150° is $\frac{1}{2}$. Now let's look at 210° which is that angle reflected across the x axis. Here the y value is $\frac{-1}{2}$ so sin 210° = $\frac{-1}{2}$. Finally let's look at 330° which is the original angle reflected across the x axis. Here the y value is $\frac{-1}{2}$ so sin 330° = $\frac{-1}{2}$.

Sample Questions:

27. The sides of the angle with measure θ are the positive x-axis and a portion of the line y = -x, as shown in the standard (x,y) coordinate plane below. What is the value of tan θ? (Hint: this is multiple choice. See if you can eliminate any possible answers. Will tan θ be positive or negative? What is the angle of y = -x? Can you tell what the measure of the angle is?)

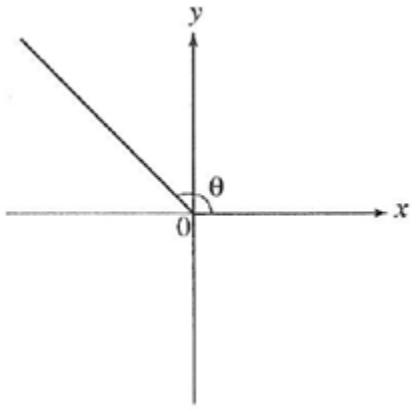
A. 1

B. $\frac{\sqrt{2}}{2}$

C. $\frac{1}{2}$

D. $\frac{-\sqrt{2}}{2}$

E. -1



28. An angle with vertex at the origin and measure θ is shown in the standard (x,y) coordinate plane below. If one side of the angle includes the positive x-axis and the other side passes through (-12, -5), then what is the sine of θ? (Hint: make a triangle and label it. If the point is [-12,-5] what are the lengths of the sides of the triangle? What is the hypotenuse?)

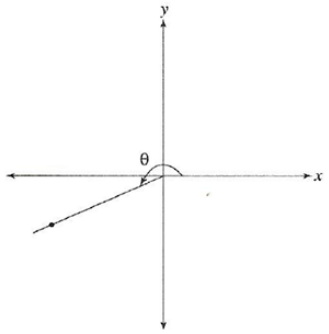
A. $\frac{-12}{5}$

B. $\frac{-12}{13}$

C. $\frac{-5}{13}$

D. $\frac{5}{12}$

E. $\frac{13}{12}$



29. For θ, an angle whose measure is between 270° and 360°, cos $\theta = \frac{12}{13.}$ Which of the following equals tan θ? (Hint: if they don't give you a figure, draw one. Make a triangle and label it. What quadrant is the triangle in? What values are positive and which are negative?)

A. $\frac{-5}{12}$

B. $\frac{-5}{13}$

C. $\frac{5}{13}$

D. $\frac{12}{5}$

E. $\frac{12}{13}$

30. If $0° < \theta < 90°$ and $\sin\theta = \frac{\sqrt{11}}{2\sqrt{3}}$, then cos θ = ? (Hint: What quadrant would this be in? If they don't give a figure, draw one. Sin = $\frac{opposite}{hypotenuse}$. From there how would you find the adjacent side?)

A. $\frac{1}{2\sqrt{3}}$

B. $\frac{1}{\sqrt{11}}$

C. $\frac{2}{\sqrt{3}}$

D. $\frac{2\sqrt{3}}{\sqrt{11}}$

E. $\frac{11}{2\sqrt{3}}$

31. If $0° < \theta < 90°$ and $\cos\theta = \frac{5\sqrt{2}}{8}$, then tan θ = ? (Hint: What quadrant would this be in? If they don't give a figure, draw one. Cos = $\frac{adjacent}{hypotenuse}$. From there how would you find the opposite side?)

A. $\frac{5}{\sqrt{7}}$

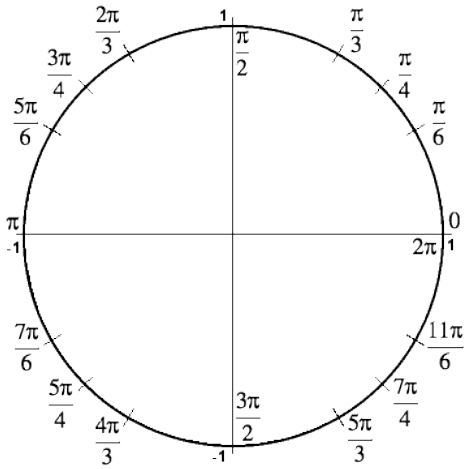
B. $\frac{\sqrt{7}}{5}$

C. $\frac{\sqrt{14}}{8}$

D. $\frac{8}{\sqrt{14}}$

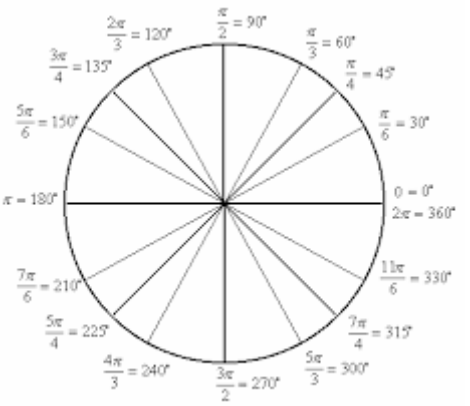
E. $\frac{8}{5\sqrt{2}}$

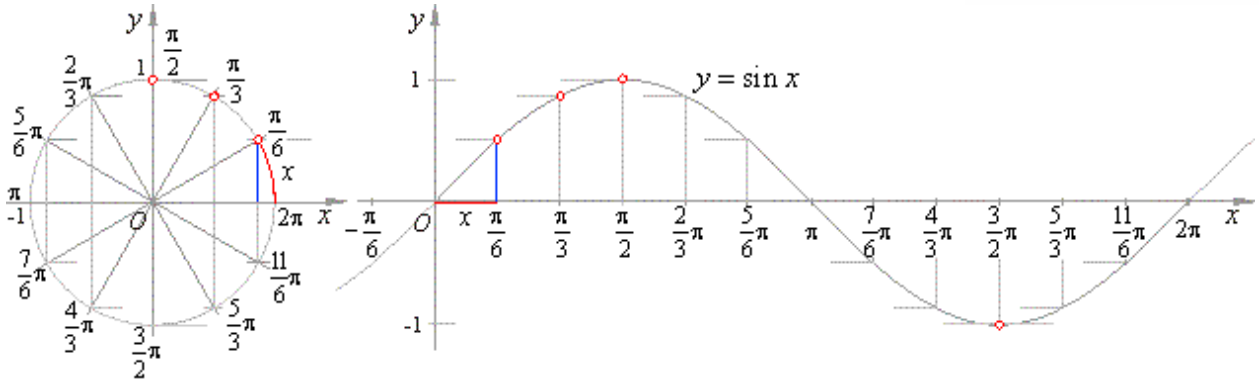
RADIANS



Radians are associated with the formula for the circumference of a circle. Circumference = $2\pi r$. In the unit circle, the radius is 1 so the formula would be circumference = $2\pi(1)$ or simply 2π. To go around a circle takes 2π radians. To go half way around the circle is π radians. To go a quarter around the circle is $\frac{\pi}{2}$ radians and so on.

When you're using your calculator to find the sine, cosine, and tangent, you need to know if you're using degrees or radians. 30° is very different from 30 radians. Usually, the button is labeled "DRG" where you choose the mode to be measured in degrees, radians or gradians on a calculator. On most calculators there are small letters near the top that indicate which mode the calculator is in. Look for the letters DEG to indicate that it is measuring in degrees, and RAD to indicate that the calculator is in radian mode. If you are in radian mode and try to find the sine using degrees you will get very wrong answers. Similarly if you are in degree mode and you try to find the sine using radians, you will get very wrong answers.





Notice the sine wave above uses radian measurements. Try finding the sine of $\frac{\pi}{2}$ using a calculator. Make sure that the calculator is in radian mode. Change the calculator to degree mode and find the sine of 90°. These answers should be the same (both answers are 1). Now try finding the sine of π using a calculator, which mode should the calculator be in? Find sine of 180°, which mode should the calculator be in? Are the answers the same?

To convert degrees to radians, multiply the angle by $\frac{\pi \text{ radians}}{180°}$

To convert radians to degrees, multiply the angle by $\frac{180°}{\pi \text{ radians}}$

Sample Questions:

32. Which of the following is the set of all values of x such that $0 \leq x \leq 2\pi$ and cos x = $\frac{-1}{2}$? (Notice that this question is using radians. The question is asking where the cos would be $\frac{-1}{2}$ which means that the x value would be $\frac{-1}{2}$. There are only 2 places on the unit circle where that happens. Since this question happens to be multiple choice, another possible way to find the answer is to check the answer choices with the calculator. Do these values have a cos of -0.5? [make sure your calculator is in radian mode])

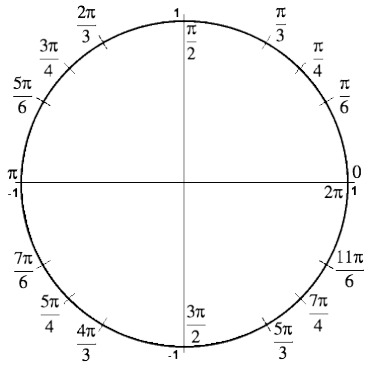
A. $\left\{\frac{\pi}{6}, \frac{5\pi}{6}\right\}$

B. $\left\{\frac{\pi}{3}, \frac{5\pi}{6}\right\}$

C. $\left\{\frac{\pi}{2}, \frac{3\pi}{6}\right\}$

D. $\left\{\frac{2\pi}{3}, \frac{4\pi}{3}\right\}$

E. $\left\{\frac{2\pi}{3}, \frac{5\pi}{3}\right\}$



33. If tan θ = - $\frac{4}{3,}$ and $\frac{\pi}{2} < \theta < \pi$, then sin θ = ? (Hint: this is in radians, what quadrant would the point be in? If they don't give a figure, draw one. Label the sides of the triangle)

A. $-\frac{4}{5}$

B. $-\frac{3}{4}$

C. $-\frac{3}{5}$

D. $\frac{3}{5}$

E. $\frac{4}{5}$

Answers

1. $\frac{3}{4}$
2. $\frac{5}{13}$
3. $\frac{5}{13}$
4. B
5. D
6. A
7. A
8. $\frac{4}{5}$
9. B
10. $\frac{3}{5}$
11. C
12. 37
13. C
14. C
15. E
16. C
17. $\frac{5}{4}$
18. $\frac{5}{3}$
19. $\frac{3}{4}$
20. $\frac{7}{25}$
21. $\frac{24}{25}$
22. $\frac{7}{24}$
23. $\frac{25}{24}$
24. $\frac{25}{7}$
25. $\frac{7}{24}$
26. E
27. C
28. A
29. A
30. B this is reduced from $\frac{\sqrt{14}}{5\sqrt{2}}$
31. D
32. E

Resources for additional practice:

Glencoe Mathematics Algebra 1 pp. 622 - 630, 636-637, 846
Glencoe Mathematics Geometry pp. 364-366
Saxon Algebra 1 pp: 796-803