Name:_____________________________________ Date:__________________________

Math 3 Proportion & Probability Part 3
Probability

MATH 3 LEVEL

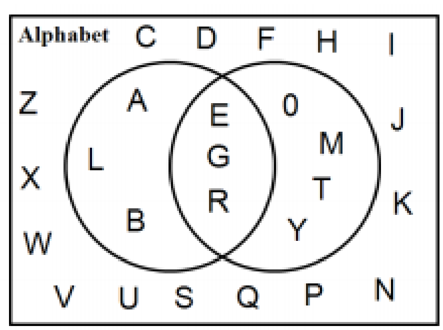
PROBABILITY

Probability is a value that represents the likelihood that an event will occur. It can be represented as a fraction, decimal or percent. A probability of zero (0) means that the event is impossible and a probability of one (1) means that the event must occur.

Events are **independent** if the occurrence of one event does not change the probability of another event occurring. Events are **dependent** if the occurrence of one changes the probability of another event occurring. For example, drawing marbles from a bag with replacement is independent, while drawing marbles from a bag without replacement is dependent. **Joint probability** is the likelihood of two or more events occurring at the same time.

Formula	**Description**	**Example**
$P(A)=\frac{\text{number of favorable outcomes}}{\text{total number of outcomes}}$	Probability of the individual event A occurring.	Flipping a coin $P(\text{heads})=\frac{1}{2}$
$P(A\cap B)=P(A)\cdot P(B)$	Joint probability of independent events.	Flipping a coin AND rolling a die $P(\text{heads and }5)=P(\text{heads})\cdot P(5)$ $=\frac{1}{2}\cdot\frac{1}{6}$ $=\frac{1}{12}$

<table>
<tr><th colspan="2">The Addition Rule</th></tr>
<tr><td>$P(A\cup B)=P(A)+P(B)-P(A\cap B)$</td><td>The addition rule finds the probability of event A occurring or event B occurring.</td></tr>
<tr><td>A letter in the word Algebra or a letter in the word Geometry. Where event A is Algebra and event B is Geometry.

$P(A\cup B)=P(A)+P(B)-P(A\cap B)$
$=\frac{6}{26}+\frac{7}{26}-\frac{3}{26}$
$=\frac{10}{26}=\frac{5}{13}$</td><td>
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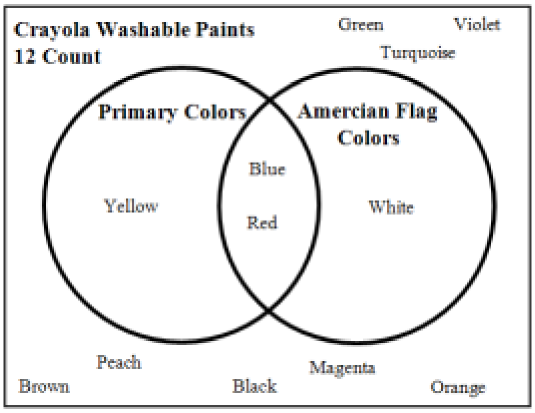
Venn Diagram Example:



P(peach)$=\frac{1}{12}$

P(color in American flag)$=\frac{3}{12}=\frac{1}{4}$

P(primary color and American flag)=
$\frac{2}{12}=\frac{1}{6}$

P(pink)$=\frac{0}{12}=0$

Two-Way Frequency Table Example:

	Curfew: Yes	Curfew: No	Total
Chores: Yes	**13**	**5**	**18**
Chores: No	**12**	**3**	**15**
Total	**25**	**8**	**33**

P(has chores)$=\frac{18}{33}$

P(doesn't have a curfew)$=\frac{8}{33}$

P(has a curfew and chores)$=\frac{13}{33}$

P(has chores $\cap$ doesn't have a curfew)$=\frac{5}{33}$

P(has a curfew)$=\frac{25}{33}$

Sample Questions:

1. You have an equally likely chance of choosing an integer from 1 to 50. Find the probability of each of the following events.
   a. An even number

   b. A number less than 35 is chosen

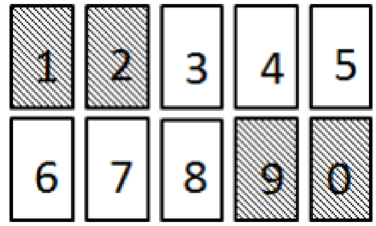
   c. A prime number is chosen

   d. A perfect cube is chosen

2. You randomly chose two marbles, replacing the first marble before drawing again, from a bag containing 10 black, 8 red, 4 white, and 6 blue marbles. Find the probability of each of the following events.
   a. A white marble, then a red marble is selected.

   b. A red marble is not selected, then a blue marble is selected.

3. Drawing a card from the cards in the diagram, determine the probability of each of the following.

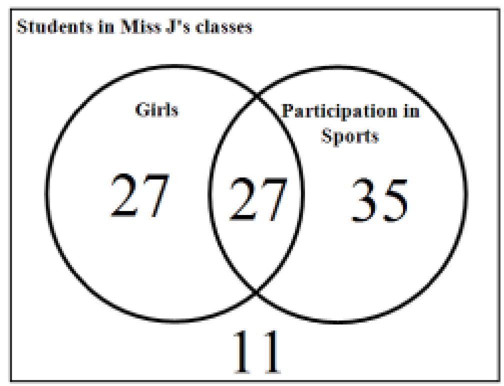


   a. P(Even or shaded)

   b. P(White or odd)

   c. P(Less than four or shaded)

4. Using the Venn diagram, answer the following questions.



   a. P(girls)

   b. P(sports, not girls)

   c. P(not sports)

5. Find the marginal totals. Then use the table to find the probabilities below.

	Brown Hair	Blonde Hair	Red Hair	Black Hair	Other Hair	Total
Male	42	11	3	17	27	
Female	47	16	13	9	15	

   a. P(male)

   b. P(red hair)

   c. P(blonde hair ∩ male)

   d. P(female ∩ not other hair)

   e. P(not female ∩ not male)

DEPENDENCE

Two events are independent if P(A) * P(B)=P(A∩B)

This table is used for the following two examples.

	10^{th}	11^{th}	12^{th}	Total
Male	320	297	215	832
Female	285	238	216	739
Total	605	535	431	1571

Example: Are being a male and being in 10^{th} grade independent?

Solution: P(male)= $\frac{832}{1571}$ and P(10th grade)= $\frac{605}{1571}$

P(male ∩10th grade)= $\frac{832}{1571} * \frac{605}{1571} \stackrel{?}{=} \frac{320}{1571}$ 0.204 = 0.204

The product of the probabilities of the individual events is equal to the probability of the intersection of the events; therefore, the events are independent.

Example: Are being a female and being in 12th grade independent?

Solution: P(female)= $\frac{739}{1571}$ and P(12th grade)= $\frac{431}{1571}$

P(female ∩12th grade)= $\frac{739}{1571} * \frac{431}{1571} \stackrel{?}{=} \frac{216}{1571}$ 0.129 ≠ 0.137

The product of the probabilities of the individual events is not equal to the probability of the intersection of the events; therefore, the events are not independent.

Sample Questions:

Determine whether or not the following events are independent.

6. If P(A)=0.7, P(B)=0.3, and P(A∩B)=0.21, are events A and B independent? Why or why not?

7. Jaron has a dozen cupcakes. Three are chocolate with white frosting, three are chocolate with yellow frosting, four are vanilla with white frosting, and two are vanilla with yellow frosting. Are cake flavor and frosting color independent?

8. This table represents the favorite leisure activities for 50 adults. Use it to answer the following questions.

	Dance	Sports	TV	Total
Men	2	10	8	
Women	16	6	8	

A. Find the probability of your gender.

B. Find the probability of your favorite leisure activity.

C. Find the probability of P(your gender ∩ your favorite leisure activity).

D. Are your gender and your favorite leisure activity independent?

CONDITIONAL PROBABILITY

A probability that takes into account a given condition is called a conditional probability. A given condition is when we already know the outcome of one of the events. For example, when flipping a coin and rolling a die, the probability of "rolling a 6 given heads", means we already know the coin has resulted in heads. This is written P(6 | heads).

The conditional probability formula is

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

Example:

A bakery sells vanilla and chocolate cupcakes with white or blue icing.

	White	Blue	Total
Vanilla	3	5	8
Chocolate	6	7	13
Total	9	12	21

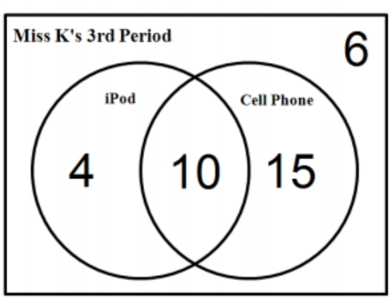
$$P(\text{Vanilla} \mid \text{Blue}) = \frac{P(\text{Vanilla} \cap \text{Blue})}{P(\text{Blue})} = \frac{5}{12}$$

$$P(\text{White} \mid \text{Chocolate}) = \frac{P(\text{White} \cap \text{Chocolate})}{P(\text{Chocolate})} = \frac{6}{13}$$

Alex's favorite cupcake is chocolate with blue icing. What is the probability he will get his favorite cupcake if all the vanilla cupcakes have already been sold?

$$P(\text{Blue} \mid \text{Chocolate}) = \frac{P(\text{Blue} \cap \text{Chocolate})}{P(\text{Chocolate})} = \frac{7}{13}$$

Example:



$$P(\text{iPod} \mid \text{Cell Phone}) = \frac{P(\text{iPod} \cap \text{Cell Phone})}{P(\text{Cell Phone})} = \frac{10}{25} = \frac{2}{5}$$

$$P(\text{Cell Phone} \mid \text{No iPod}) = \frac{P(\text{Cell Phone} \cap \text{No iPod})}{P(\text{No iPod})} = \frac{15}{15+6} = \frac{15}{21} = \frac{5}{7}$$

Miss K finds an iPod after class. What is the probability the owner has an iPod and no cell phone?

$$P(\text{iPod} \mid \text{No Cell Phone}) = \frac{P(\text{iPod} \cap \text{No Cell Phone})}{P(\text{No Cell Phone})} = \frac{4}{4+6} = \frac{4}{10} = \frac{2}{5}$$

Sample Questions:

9. Use this table to answer the following questions.

	Bus	Car	Walk	Total
Male	146	166	82	
Female	154	185	64	
Total				

a. P(Bus | Male)

b. P(Female | Doesn't Walk)

c. What is the probability that Melissa rides the bus? Write the conditional probability equation and then find the probability.

d. Jordan walks to school. What is the probability Jordan is male? Write the conditional probability equation and then find the probability.

10. Use this diagram to answer the following questions.

a. P(After School Job | Male)

b. P(Female | No After School Job)

c. Is the probability of having an after school job given you are male the same as the probability of being male given that you have an after school job? Use probabilities to justify your answer.

d. A student works at Bob's Taco Shop what is the probability the student is female?

INDEPENDENT

Events A and B are independent if and only if they satisfy the probability A given B equals the probability of A OR the probability of B given A equals the probability of B.

$$P(A|B) = P(A) \quad \text{or} \quad P(B|A) = P(B)$$

Example:

A bakery sells vanilla and chocolate cupcakes with white or blue icing.

	White	Blue	Total
Vanilla	3	5	8
Chocolate	6	7	13
Total	9	12	21

Are color of icing and cupcake flavor independent?

$$P(Blue|Vanilla) \stackrel{?}{=} P(Blue)$$

$$\frac{P(Blue \cap Vanilla)}{P(Vanilla)} \stackrel{?}{=} P(Blue)$$

$$\frac{5}{8} \stackrel{?}{=} \frac{12}{21}$$

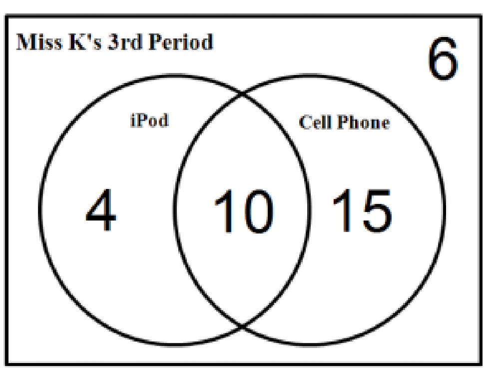
$$0.625 \neq 0.571$$

Therefore the color of icing and cupcake flavor are **not** independent.

Note: Keep in mind the above can also be tested using any of the following options.

- P (Blue | Chocolate) $\stackrel{?}{=}$ P(Blue)
- P (Vanilla | Blue) $\stackrel{?}{=}$ P(Vanilla)
- P (Chocolate | Blue) $\stackrel{?}{=}$ P(Chocolate)
- P (White | Chocolate) $\stackrel{?}{=}$ P(White)
- P (White | Vanilla) $\stackrel{?}{=}$ P(White)
- P (Vanilla | White) $\stackrel{?}{=}$ P(Vanilla)
- P (Chocolate | White) $\stackrel{?}{=}$ P(Chocolate)

Example:



Are having an iPod and having a cell phone independent?

$$P(\text{iPod}|\text{Cell Phone}) \stackrel{?}{=} P(\text{iPod})$$

$$\frac{P(\text{iPod} \cap \text{Cell Phone})}{P(\text{Cell Phone})} \stackrel{?}{=} P(\text{iPod})$$

$$\frac{10}{25} \stackrel{?}{=} \frac{14}{35}$$

$$\frac{2}{5} = \frac{2}{5}$$

Therefore, having an iPod and having a cell phone are independent.

Sample Questions:

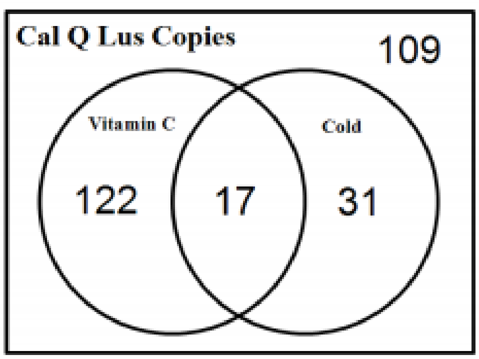
11. Students were asked what their main goal for their high school years was. The reported goals were getting good grades, being popular, or excelling at sports.

	Goals			
	Grades	Popular	Sports	Total
Boy	117	50	60	
Girl	130	91	30	

a. Is the probability of having good grades as a goal independent of gender?

b. Is gender independent of having popularity as a goal?

12. Workers at Cell phone store were polled to see if Vitamin C was a way to reduce the likelihood of getting a cold. According to the diagram, are you less likely to catch a cold if you are taking vitamin C? Justify your answer using conditional probability.



13. Real estate ads suggest that 64% of homes for sale have garages, 21% have swimming pools, and 17% have both features. Are having a garage and having a pool independent events? Justify your answer using conditional probability.

Answer Key

1. .
   a. 25/50 = ½
   b. 34/50 = 17/25
   c. 15/50 = 3/10
   d. 3/50
2. .
   a. 2/49
   b. 15/98
3. .
   a. 7/10
   b. 8/10 = 4/5
   c. 5/10 = 1/2
4. .
   a. 54/100 = 27/50
   b. 35/100 = 7/20
   c. 38/100 = 19/50
5. .
   a. 100/200 = 1/2
   b. 2/25
   c. 11/200
   d. 85/200 = 17/40
   e. 0
6. Yes because P(A) * P(B) = 0.7(0.3) = P(A∩B) = 0.21
7. Answers may vary
   P(chocolate cake) * P(yellow frosting) $\stackrel{?}{=}$ P(chocolate cake ∩ yellow frosting)
   6/12 * 5/12 $\stackrel{?}{=}$ 3/12
   0.5 * 0.416666 = 0.21 $\neq$ 0.25
   Therefore, cake flavor and frosting color are not independent
8. .

	Dance	Sports	TV	Total
Men	1/25	1/5	4/25	2/5
Women	8/25	3/25	4/25	3/5
	9/25	8/25	8/25	

   a. Answers will vary—see chart
   b. Answers will vary—see chart
   c. Answers will vary—see chart
   d. None of the choices are independent
9. .
   a. 146/394 = 73/197
   b. 339/651 = 113/219
   c. 154/403 = 22/29
   d. 82/146 = 41/73
10.
   a. 0.5625
   b. 0.475
   c. No, one is 0.5625 and the other is 0.45
   d. 0.55
11. .
   a. Yes, $\frac{P(grades \cap male)}{P(male)}$ = P(grades) = 0.52
   b. No, $\frac{P(populatiry \cap female)}{P(female)} \neq$ P(popularity)
   91/251 $\neq$ 141/478

12. Yes, P(cold | Vitamin C) = 0.122 while
   P(cold | no Vitamin C) = 0.221

13. P(garage | swimming pool) $\stackrel{?}{=}$ P(garage)
   OR
   P(swimming pool | garage) $\stackrel{?}{=}$ P(swimming pool)

   17/(21+17) $\stackrel{?}{=}$ (64+17)/102
   0.447 $\neq$ 0.79
   OR
   17/(64+17) $\stackrel{?}{=}$ (21+17)/102
   0.21 $\neq$ 0.37