Name:___________________________________ Date:_________________________

Math 3 Coordinate Geometry Part 1
Review & Graphs

MATH 1 REVIEW:
SLOPE-INTERCEPT FORM **y = mx + b** (where m represents the slope and b represents the y-intercept)

1. What is the slope of the line y = $\frac{-1}{4}$X + 3?

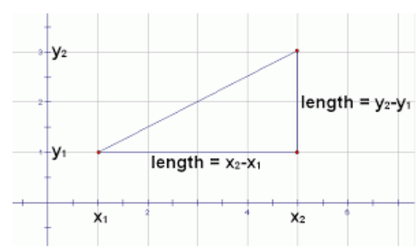
2. At what point does the line y = $\frac{2}{9}$X - 1 cross the y axis?

RE-WRITING EQUATIONS IN SLOPE-INTERCEPT FORM
To find the slope or y-intercept of a line that is not in slope-intercept form, re-write the equation in slope-intercept form so it looks like y = mx + b.

3. At which y-coordinate does the line described by the equation 6y - 3x = 18 intersect the y-axis?

4. What is the slope of the line described by the equation 2y - 5x = 8?

DISTANCE BETWEEN TWO POINTS
DISTANCE FORMULA $\boldsymbol{d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$
This formula is derived from the Pythagorean theorem. The difference between x_1 and x_2 is one leg and the difference between y_1 and y_2 is the other leg, and the distance between the two points is the hypotenuse.



5. Point M (-1, 3) and point N (6, 5) are points on the coordinate plane. What is the length of the segment MN?

6. Points A and B lie in the standard (x,y) coordinate plane. The (x,y) coordinates of A are (7, 1), and the (x,y) coordinates of B are (-2,-4). What is the distance from A to B?

FINDING THE SLOPE FROM TWO POINTS

SLOPE = m = $\frac{change\ in\ y}{change\ in\ x}$ = $\frac{rise}{run}$ = $\frac{y_2 - y_1}{x_2 - x_1}$

7. What is the slope of the line that contains the points J (2,3) and K (0,-2)?

8. What is the slope of the line that contains the points H (-9, 1) and G (2, -7)?

CREATING AN EQUATION FOR A LINE FROM TWO POINTS

Finding an equation for a line when we are only given 2 points takes 2 steps. We know that y = mx + b, where m is the slope and b is the y -intercept. So in order to create the equation we need m and we need b.

<u>Step 1: find m (slope)</u> slope = m = $\frac{change\ in\ y}{change\ in\ x}$ = $\frac{rise}{run}$ = $\frac{y_2 - y_1}{x_2 - x_1}$

<u>Step 2: find b (y-intercept)</u> Use the formula y = mx + b to solve for b. Plug in the slope you found in step 1 for m, then choose one of the points that was given (it doesn't matter which one), plug in the x value for x in the formula and the y value in for y in the formula. Solve for b.

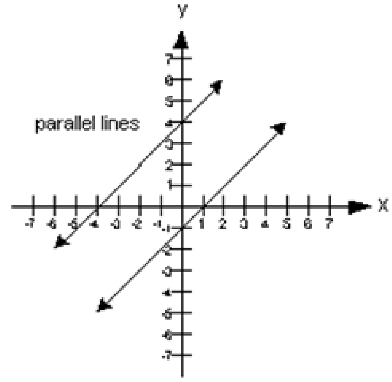
9. What is the equation of the line that passes through points A (-8,3) and B (0,-1)?

10. What is the equation of the line that passes through the points (0,0) and (6,3)?

11. What is the equation of the line that passes through points Z (4,7) and W (1, 3)?

PARELLEL LINES ON THE COORDINATE PLANE

Parallel lines always have the same slope. Therefore, if two lines have the same slope then they are parallel. In systems of equations, parallel lines have no solutions. (Since 'solutions' are where the lines cross and parallel lines do not cross).

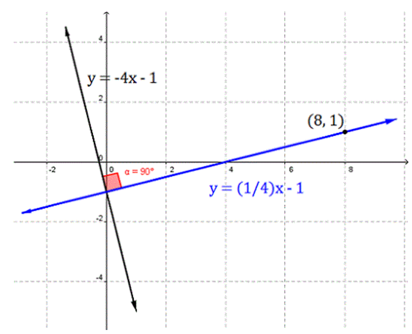


12. Write the equation of a line which is parallel to the line y = 2x + 3 and passes through the point (0, -1).

13. Which of the following systems of equations does NOT have a solution
 A. x + 3y = 19 and 3x + y = 6
 B. x + 3y = 19 and x - 3y = 13
 C. x - 3y = 19 and 3x - y = 7
 D. x - 3y = 19 and 3x + y = 6
 E. x + 3y = 6 and 3x + 9y = 7

PERPENDICULAR LINES ON THE COORDINATE PLANE

When two lines are perpendicular, the slope of one is the negative reciprocal of the other. If the slope of one line is m, the slope of the other is $\frac{-1}{m}$.



14. Write the equation of a line which is perpendicular to the line y = $\frac{2}{3}$x + 3 and passes through the point (0, -1)

15. Line q passes through the point (-2, -3) in the standard (x,y) coordinate plane and is perpendicular to the line described by the equation y = −2x + 4. What is the equation that describes line q?

MIDPOINTS BETWEEN TWO POINTS ON THE COORDINATE PLANE

If you need to find the point that is exactly halfway between two given points, just average the x-values and the y-values.

Midpoint formula: $\left(\frac{x_1+x_2}{2}, \ \frac{y_1+y_2}{2}\right)$

For example: find the midpoint between (–1, 2) and (3, –6).

$\left(\frac{x_1+x_2}{2}, \ \frac{y_1+y_2}{2}\right) = \left(\frac{-1 \ +3}{2}, \ \frac{2 \ +-6}{2}\right) = \left(\frac{2}{2}, \ \frac{-4}{2}\right)$ = (1, -2)

16. In the standard (x,y) coordinate plane, points P and Q have coordinates (2,3) and (12,-15), respectively. What are the coordinate of the midpoint of PQ?

17. The midpoint of line segment AC in the standard (x,y) coordinate plane has coordinates (4,8). The (x,y) coordinates of A and C are (4,2) and (4, Z), respectively. What is the value of Z?

MATH 2 REVIEW:
EQUATION OF A CIRCLE

The equation for a circle of radius r and centered at (h,k) is: $(x-h)^2+(y-k)^2=r^2$

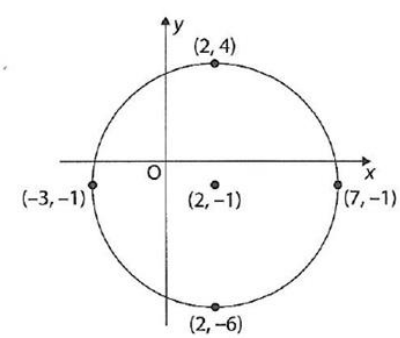
Using the example of the equation $(x-2)^2$ + $(y+1)^2$ = 25.
Since the equation of a circle: $(x-h)^2$ + $(y-k)^2$ = r^2 we can compare this to the given equation.
$(x-h)^2$ + $(y-k)^2$ = r^2
$(x-2)^2$ + $(y+1)^2$ = 25

h (or the x value of the center of the circle) must be 2 since I see $(x-2)^2$

k (or the y value of the center of the circle) must be -1 since I see $(y+1)^2$. This one is a little trickier, since I need to remember that it is y - k and since I see y + 1 that means I need a negative value for k since subtracting a negative value is the same as adding. $(y-[-1])$ = $(y+1)$.

The radius must be 5 since r^2 = 25.

Putting all this together I know that the center of the circle is at the point (2, -1) and the radius is 5. With this information I can also figure out other things about the circle like the diameter, circumference, and area of the circle. The diagram below shows the graph of the circle with center at point (2,-1) and a radius of 5.



18. The equation of a circle in the standard (x,y) coordinate plane is given by the equation $(x-2)^2$ + $(y+4)^2$ = 16. What is the center of the circle?

19. The equation of a circle in the standard (x,y) coordinate plane is given by the equation $(x)^2$ + $(y)^2$ = 12. What is the center of the circle?

20. The equation of a circle in the standard (x,y) coordinate plane is given by the equation $(x-3)^2$ + $(y-2)^2$ = 36. What is the radius of the circle?

21. A circle with the equation x^2 + y^2 = 49 is graphed in the standard (x,y) coordinate plane. At which 2 points does this circle intersect the x-axis? (Hint: Where is the center of the circle? What is the radius of the circle? Would it help to draw a graph?)
   A. (-1, 0) and (1, 0)
   B. (-7, 0) and (7, 0)
   C. (-14, 0) and (14, 0)
   D. (-21, 0) and (21, 0)
   E. (-49, 0) and (49, 0)

22. In the standard (x,y) coordinate plane, what is the area of the circle $(x-3)^2$ + $(y+2)^2$ = 25?
   A. 5π
   B. 10π
   C. 25π
   D. 125π
   E. 225π

EQUATION OF PARABOLAS

The graph of an equation in the form $\boldsymbol{y = ax^2 + bx + c}$ is a parabola. A parabola has a line of symmetry. The point on the parabola that is on its line of symmetry is called the vertex. We can find the x value of the vertex with the formula: $\frac{-\boldsymbol{b}}{\boldsymbol{2a}}$, then by plugging the x value into the equation, we can find the y value of the vertex.

For example to find the vertex of the parabola $y = x^2 - 2x - 3$ we compare it to the formula $y = ax^2 + bx + c$

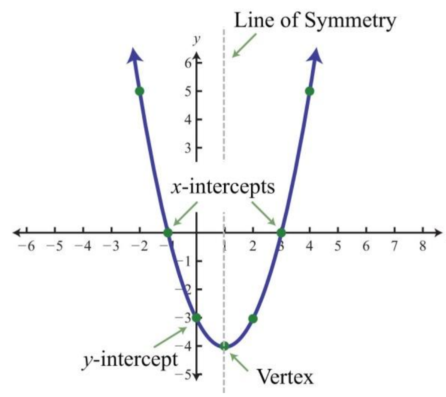
$y = x^2 - 2x - 3$ comparing the two formulas I can see that a = 1, b = -2 and c = -3.

Then use the formula $\frac{-b}{2a}$ and plug in the values for a and b. $\frac{-(-2)}{2(1)} = \frac{2}{2} = 1$.

So the x value of the vertex is 1. Then I plug that into the original equation $y = x^2 - 2x - 3$

$y = (1)^2 - 2(1) - 3$ $(1)^2$ = -4 so the y value of the vertex is -4.

So the vertex of the equation $y = x^2 - 2x - 3$ is (1, -4)



23. What is the vertex of the parabola $y = 3x^2 + 6x + 2$?

24. What is the vertex of the parabola $y = x^2 + 6x - 5$

VERTEX FORM OF PARABOLA

Since the vertex is the most important point on the parabola there is another form for writing an equation of a parabola called the vertex form. It looks like this:

$y = a(x - h)^2 + k$. Where (h,k) is the vertex.

For example to find the vertex of the parabola described by the equation $y = -2(x - 1)^2 + 2$ we can compare it to the vertex form of a parabola.

$y = a(x - h)^2 + k$ vertex form of a parabola

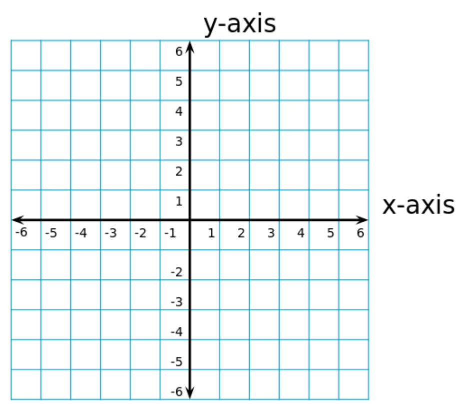
$y = -2(x - 1)^2 + 2$ from this I can see that a = -2, h = 1, and k = 2. Since the vertex is just (h,k) the vertex of this parabola must be the point (1, 2). Done.

25. What is the vertex of the parabola $y = (x-4)^2 + 5$?

26. What is the vertex of the parabola $y = 4(x+1)^2 - 3$?

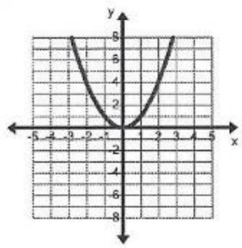
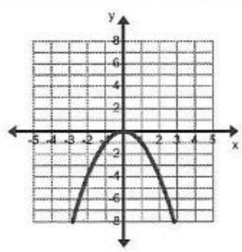
27. What is the vertex of the parabola $y = -1(x-3)^2 + 7$?

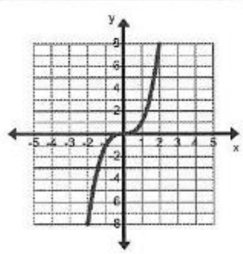
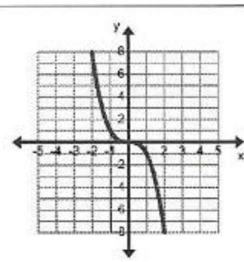
28. Graph the parabola $y = \frac{1}{4}(x)^2 - 1$. (Hint: find the vertex, then select a few values for x and find the corresponding y value. Graph enough points until you know the basic shape of the parabola.)



MATH 3:
POWER FUNCTION GRAPHS AND END BEHAVIOR

We're going to examine the "end behavior" of various power functions. End behavior refers to the directions that the left end and right end of the function are headed. A quadratic equation is a 2nd degree function, meaning the variable x is raised to the 2nd power (x^2). As we've seen, quadratic equations create parabolas. A quadratic equation with a positive coefficient creates a "smile" shape with both ends reaching toward positive infinity. A quadratic equation with a negative coefficient creates a "frown" shape with both ends reaching toward negative infinity.

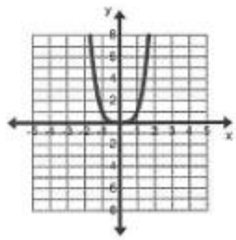
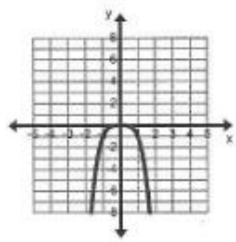
Function	Graph	Left End Behavior	Right End Behavior
$f(x)=x^2$		$\lim_{x\to-\infty} f(x)=+\infty$	$\lim_{x\to\infty} f(x)=+\infty$
$f(x)=-x^2$		$\lim_{x\to-\infty} f(x)=-\infty$	$\lim_{x\to\infty} f(x)=-\infty$

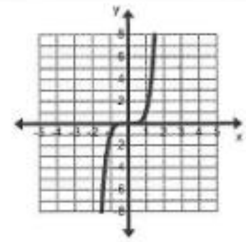
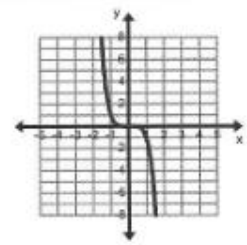
Function	Graph	Left End Behavior	Right End Behavior
$f(x)=x^3$		$\lim_{x\to-\infty} f(x)=\ ?$	$\lim_{x\to\infty} f(x)=\ ?$
$f(x)=-x^3$		$\lim_{x\to-\infty} f(x)=\ ?$	$\lim_{x\to\infty} f(x)=\ ?$

Sample Questions:

29. Describe the left and right end behavior of the positive cubic function (x^3) graphed above.

30. Describe the left and right end behavior of the negative cubic function ($-x^3$) graphed above.

Function	Graph
$f(x)=x^4$	
$f(x)=-x^4$	

Function	Graph
$f(x)=x^5$	
$f(x)=-x^5$	

Sample Questions:

31. Refer to the graph above. Does the quartic (4th degree or x^4) graph more closely resemble $f(x) = x^2$ or $f(x) = x^3$?

32. Refer to the graph above. Does the quintic (5th degree or x^5) graph more closely resemble $f(x) = x^2$ or $f(x) = x^3$?

By noticing patterns, we can generalize the end behavior of any polynomial. For any polynomial

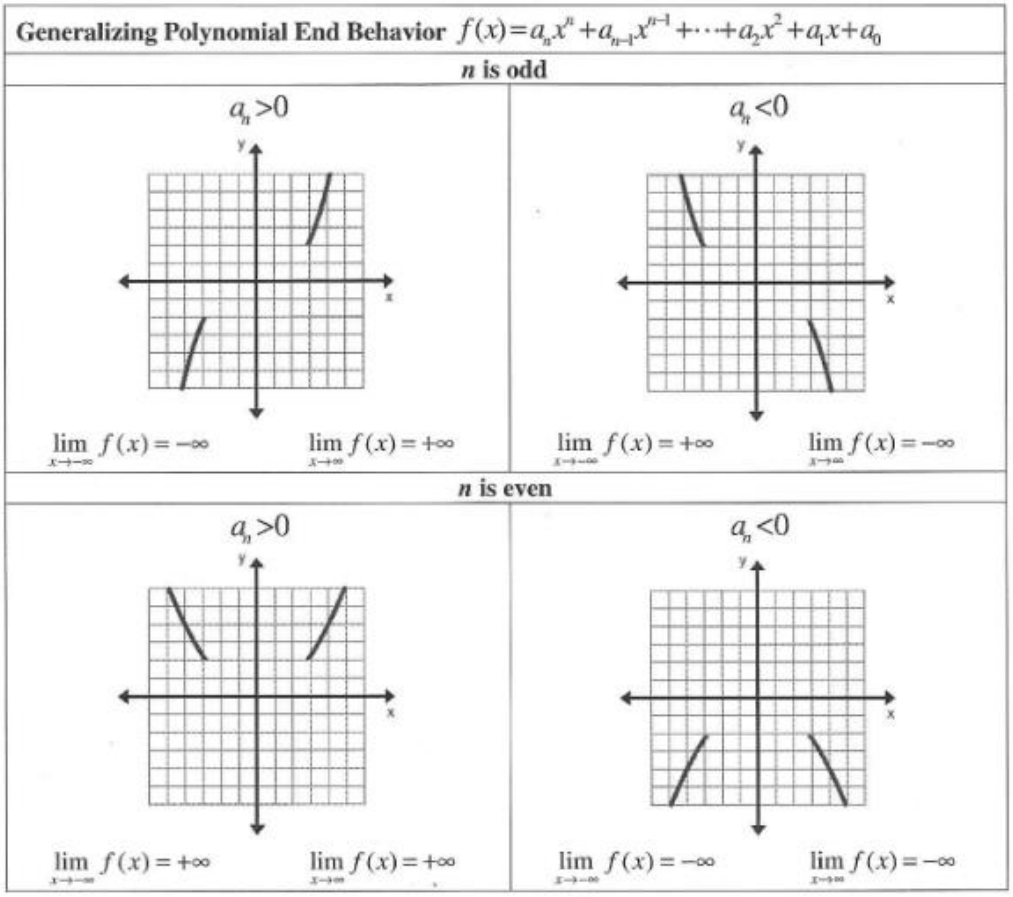
$$f(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_2 x^2 \; a_1 x \quad + a_0$$

we only need to look at the first term $a_n x^n$. We're looking for two things: first, is the exponent odd or even?

Second, is the coefficient positive (> 0) or negative (< 0)? If the exponent is even then the ends will be like those of a parabola, an upturned smile if the coefficient is positive (towards $+\infty$).

Contrariwise the edges will be like a downturned frown if the coefficient is negative (towards $-\infty$).

Likewise, if the exponent is odd then one end will turn upward and the other downward. If the coefficient is positive then the left end points down (towards $-\infty$) and the right end points up (towards $+\infty$). If the coefficient is negative then left moves toward $+\infty$ and the right moves toward $-\infty$.



Sample Questions:

Refer to the polynomial $f(x) = -x^3 + 6x^2 - 5x + 7$ for questions 33-35.

33. Is the exponent of the first term odd or even?

34. Is the coefficient of the first term positive or negative?

35. Describe the left and right end behavior for the polynomial

Without graphing, determine if the function is odd or even and describe the end behavior of each polynomial

36. $f(x) = 2x^5 + 7x^3 - 6x$

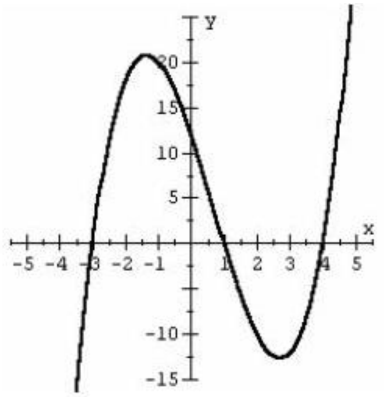
37. $f(x) = -3x^4 + x^3 - 5x + 9$

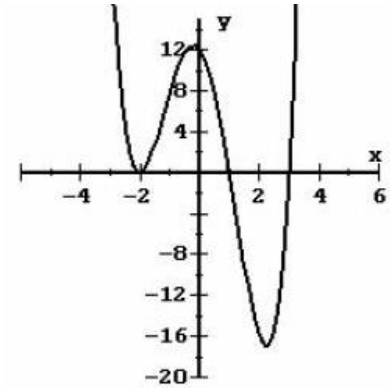
38. $f(x) = -x^7 + 2x^6 + 4x^5 + 3x^4 - x^3$

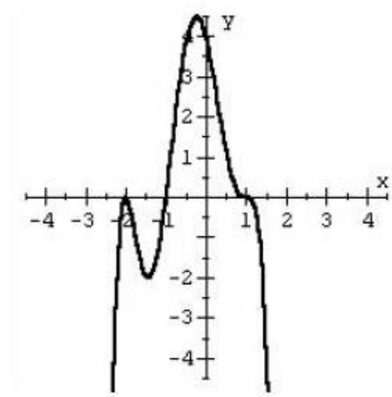
39. $f(x) = 5x^8 - 12x^6 + x^5 - 3x^3 - x + 4$

ZEROS

The "zeros" of a graph are all the points where the function crosses or touches the x-axis. This will happen each time y = 0.



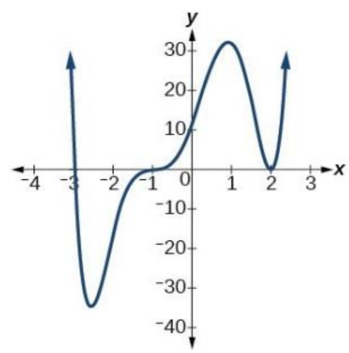




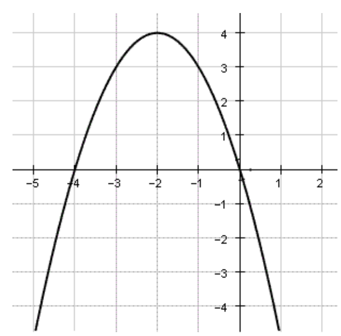
In the graph above and to the left, the zeros are at x = -3, x = 1 and x = 4. In the graph above and to the center the zeros are at x = -2, x = 1, and x = 3. In the graph above and to the right the zeros are at x = -2, x = -1, and x = 1.

Sample Questions:

40. What are the zeros of the function?



41. What are the zeros of this function?



FUNDAMENTAL THEOREM OF ALGEBRA

For any polynomial $f(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_2x^2\ a_1x \quad + a_0$ we can determine how many zeros the function will have by looking at the first term. The number of zeros will be the same as the highest exponent. In other words: a polynomial function of degree n> 0 has n complex zeros (every real number is a complex number i.e., 3 = 3 + 0i) Some of the zeros may be repeated.

Sample Questions:

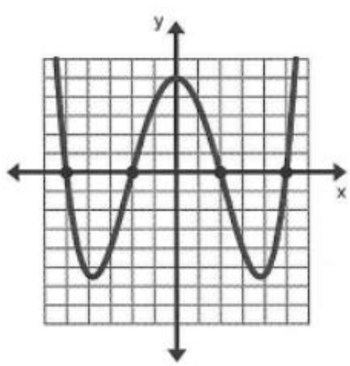
For questions 42-43 determine the number of zeros each polynomial has.

42. $f(x) = 3x^3 + 2x^2 - 7x + 2$

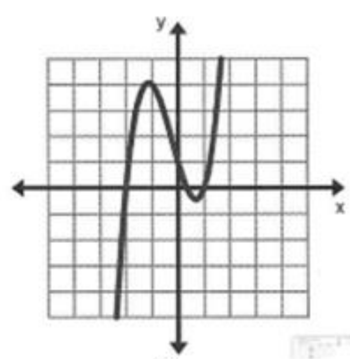
43. $g(x) = x^4 - 29x^2 + 100$

44. Using the equation from question 42 above [$f(x) = 3x^3 + 2x^2 - 7x + 2$] which graph below might represent this function? (A or B)

45. Using the equation from question 43 above [$g(x) = x^4 - 29x^2 + 100$] which graph below might represent this function (A or B).



A



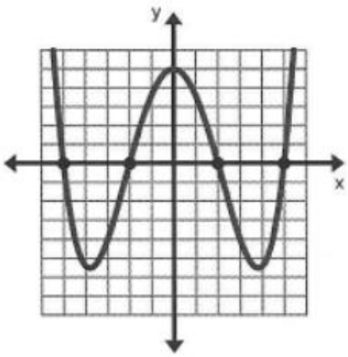
B

SOLUTIONS OR ROOTS OF QUADRATIC EQUATIONS

A real number x will be called a **solution** or a **root** if it satisfies the equation. It is easy to see that the **roots** are exactly the x-intercepts of the quadratic function, that is the intersection between the graph of the quadratic function with the x-axis. Therefore, the solutions, or roots, are the zeros described above.

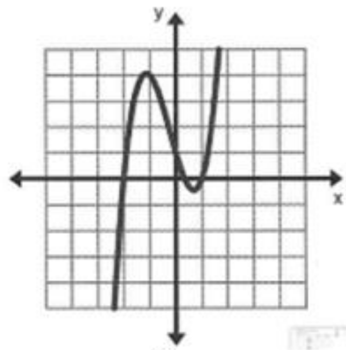
Sample Questions:

46. Graph A has how many roots or solutions?



A

47. Graph B has how many roots or solutions?



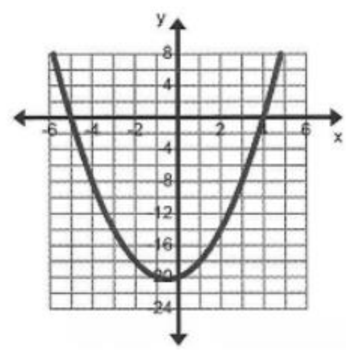
B

We can find the roots or solutions by looking at a graph, we can also find roots or solutions by looking at the equation.

For example to find the solutions or roots of the function $f(x) = x^2 - x - 20$, the first step is to factor the equation and rewrite it like this $f(x) = (x-4)(x+5)$. Written this way it is easy to find the zeros (i.e. where $f(x) = 0$)

$f(x) = (x-4)(x+5)$ = 0 if x = 4, or if x = -5

The graph is shown at right.



Sample Questions:

48. Find the zeros of this polynomial: $f(x) = -(x-3)(x+2)(x-1)$

49. Find the zeros $f(x) = (x+9)(x-10)(x-6)(x+2)$

50. Find the zeros of this polynomial: $f(x) = x(x-1)(x+1)(x+5)$

51. Find the zeros of this polynomial: $f(x) = (x-4)(x-4)(x-4)$

MULTIPLICITY OF ZEROS

Sometimes a zero is repeated, like in question 51 above. There are three answers but they are all the same. This is called a "multiplicity of zeros." In the equation $f(x) = (x-4)(x-4)(x-4)$ the answer is 4 with a multiplicity of 3 since it is repeated three times.

Multiplicity of zeros messes with the graph somewhat. For example, the equation

$f(x) = (x-4)(x-4)(x-4)$ multiplied out is $f(x) = x^3 - 12x^2 + 48x - 64$,

from this equation we can see that there should be 3 zeros or places where the graph crosses or touches the x axis, but since all three answers are the same, the graph only crosses the axis once.

Sample Questions:

52. Does this function have a multiplicity of zeros? $f(x) = (x+1)(x-3)(x-1)$?

53. Does this function have a multiplicity of zeros? $f(x) = (x-5)(x-5)(x-2)$?

54. Does this function have a multiplicity of zeros? $f(x) = (x+3)(x-7)(x+3)$?

55. Does this function have a multiplicity of zeros? $f(x) = (x+1)(x-4)(x+4)(x-1)$?

Answers

1. $\frac{-1}{4}$
2. -1
3. 3
4. $\frac{5}{2}$
5. $\sqrt{53}$ or 7.28
6. $\sqrt{106}$ or 10.296
7. $\frac{5}{2}$
8. - $\frac{8}{11}$
9. y = - $\frac{1}{2}$x -1
10. y = $\frac{1}{2}$x
11. y = $\frac{4}{3}$x + $\frac{5}{3}$
12. y = 2x - 1
13. E
14. y = $\frac{-3}{2}$x -1
15. y = $\frac{1}{2}$x - 2
16. (7, -6)
17. 14
18. (2, -4)
19. (0,0)
20. 6
21. B
22. C
23. (-1, -1)
24. (-3, -14)
25. (4, 5)
26. (-1, -3)
27. (3, 7)
28. see graph at right
29. left -∞, right ∞
30. left ∞, right -∞
31. $f(x) = x^2$
32. $f(x) = x^3$
33. odd
34. negative
35. left ∞, right - ∞
36. odd, left -∞, right ∞
37. even, left -∞, right -∞
38. odd, left ∞, right - ∞
39. even, left ∞, right ∞
40. -3, -1, 2
41. -4, 0
42. 3
43. 4
44. B
45. A
46. 4
47. 3
48. 3, -2, 1
49. -9, 10, 6, -2
50. 0, 1, -1, -5
51. 4
52. no
53. yes
54. yes
55. no