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## Math 2 Variable Manipulation Part 1 Algebraic Equations

## PRE ALGEBRA REVIEW OF INTEGERS (NEGATIVE NUMBERS)

Numbers can be positive (+) or negative (-). If a number has no sign it usually means that it is a positive number. For example: 5 is really +5

## Adding Positive Numbers

Adding positive numbers is just simple addition.
Example: $2+3=5$
Solution: This equation is really saying "Positive 2 plus Positive 3 equals Positive 5"
Write it as $(+2)+(+3)=(+5)$

## Subtracting Positive Numbers

Subtracting positive numbers is just simple subtraction.
Example: 6-3=3
Solution: This equation is really saying "Positive 6 minus Positive 3 equals Positive 3"
Write it as $(+6)-(+3)=(+3)$


Subtracting a Negative is the same as Adding

Example: What is $6-(-3)$ ?
Solution: $6-(-3)=6+3=9$


Example: What is $5+(-2)$ ?
Solution: $+(-)$ are unlike signs (they are not the same), so they become a negative sign.
$5+(-2)=5-2=3$
Example: What is $25-(-4)$ ?
Solution: -(-) are like signs, so they become a positive sign.
$25-(-4)=25+4=29$
Example: What is $-6+(+3)$ ?
Solution: $+(+)$ are like signs, so they become a positive sign.
$-6+(+3)=-6+3=-3$

## Multiplying with integers

You also have to pay attention to the signs when you multiply and divide. There are two simple rules to remember:

1) When you multiply a negative number by a positive number then the product is always negative.
2) When you multiply two negative numbers or two positive numbers then the product is always positive. This is similar to the rule for adding and subtracting: two minus signs become a plus, while a plus and a minus become a minus. In multiplication and division, however, you calculate the result as if there were no minus signs and then look at the signs to determine whether your result is positive or negative.

Example: 3•(-4)
Solution: $3 \cdot(-4)=-12$
3 times 4 equals 12. Since there is one positive and one negative number, the product is negative 12.
Example: (-3)•(-4)
Solution: $(-3) \cdot(-4)=12$
Now we have two negative numbers, so the result is positive.

## Dividing Negative Numbers

Rules when dividing by negative numbers:

1) When you divide a negative number by a positive number then the quotient is negative.
2) When you divide a positive number by a negative number then the quotient is also negative.
3) When you divide two negative numbers then the quotient is positive.

Example: $\frac{-12}{3}=$ ?
Solution: $\frac{-12}{3}=-4$

Example: $\frac{12}{-3}=$ ?
Solution: $\frac{12}{-3}=-4$

Example: $\frac{-12}{-3}=$ ?
Solution: $\frac{-12}{-3}=4$

## MATH 1 REVIEW

## VOCABULARY FOR ALGEBRAIC EQUATIONS

Variable: Symbols are used to represent unspecified numbers or values. Any letter may be used as a variable.
$0.45 r$
$3 x+5$
$2+\frac{Y}{3}$
v* w
$\frac{1}{2} p q \div 4 r s$

Constant: A term whose value does not change.
$5 x+4$ (4 is the constant)

Coefficient: The numerical factor of a term.
$6 x$ (6 is the coefficient)

Term: May be a number, a variable, or a product or quotient of numbers and variables.
$3 x-5=-7 x+10$ (This equation has 4 terms: $3 x,-5,-7 x$, and +10 )

Like Terms: Terms that contain the same variables, with corresponding variables having the same power.
$2 x^{3} y^{2}$ and $-4 x^{3} y^{2}$ are like terms

Algebraic Expression: Consists of sums and/or products of numbers and variables. A variable or combination of variables, numbers, and symbols that represents a mathematical relationship.

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\frac{3 x}{2}+5-x^{2}
$$

Equation: A mathematical statement that contains an equal sign.
$9 x-8=22-x$

Solution: A set of elements that make an equation or inequality true. Any and all value(s) of the variable(s) which satisfies an equation, or inequality.

The only solution for the equation $2 x-15=-3$ is $x=4$.

## DISTRIBUTIVE PROPERTY

The distributive property is one of the most frequently used properties in math. It lets you multiply a sum by multiplying each addend separately and then add the products. It is easier to understand the meaning by looking at the examples below.


Consider the first example, the distributive property lets you "distribute" the 5 to both the 'x' and the ' 2 '.
Example: $5(\mathrm{n}+6)$
Solution: Multiply 5 by both $n$ and 6
5( $\mathrm{n}+6$ )
$5^{*} \mathrm{n}+5^{*} 6$
$5 n+30$

## ADDING AND SUBTRACTING MONOMIALS \{COMBINING LIKE TERMS)

Combining like terms is just a way to count the number of items that are the same. Another way is to keep the variable part unchanged while adding or subtracting the coefficients (numbers):
Example: $2 \mathrm{a}+3 \mathrm{a}$
Solution 1: There are 2 a's and another 3 a's, so $2+3=5$ so there are a total of 5 a's
Example: $2 \mathrm{a}+3 \mathrm{a}$
Solution 2: Add the coefficients and keep the variables the same. $2 \mathrm{a}+3 \mathrm{a}=(2+3) \mathrm{a}=5 \mathrm{a}$

Example: For all $x, 13-2(x+5)=$ ?
Solution: First distribute the -2 and then combine like terms
$13-2(x+5)$
13-2x-10
13-10-2x
$3-2 x$

## MULTI-STEP EQUATIONS

The steps to solving a multi-step equation are:

1. Distribute
2. Collect like terms
3. Gather all the variables on one side
4. Gather all the constants on other side
5. The variable should have a coefficient of 1
6. Check your answer

Example: $p-1=5 p+3 p-8$
Solution: Add $5 p$ and $3 p$ to get $8 p$. Subtract $p$ from both sides and add 8 to both sides. Then divide both sides by 7 .
$p-1=5 p+3 p-8$
$p-1=8 p-8$
-p -p
$-1=7 p-8$
$+8 \quad+8$
$\underline{7}=\underline{7 p}$
77
$1=p$

Example: $12=-4(-6 x-3)$
Solution: Distribute -4 into the parentheses. Subtract 12 from each side of the equation. Divide both sides of the equations by 24.
$12=-4(-6-3)$
$12=24 \mathrm{x}+12$
$-12 \quad-12$
$\underline{0}=\underline{24 x}$
2424
$0=x$
Zero can be the answer to an equation (as seen in the example above). It is also possible to have "No Solution". This happens when both sides can NEVER be equal.

Example: $4 m-4=4 m$
Solution: Subtract 4 m from both sides and you get $-4=0$ which is not true. $-4 \neq 0$ so there is No Solution.

A negative outside the parentheses means to distribute the negative, or a negative 1 , into the parentheses. If the variable is by itself, but is negative, divide both sides by negative 1 to get a positive variable.

Example: $14=-(p-8)$
Solution: Distribute the negative (or negative 1) into the parentheses. Subtract 8 from both sides. Divide both sides by -1 .
$14=-(p-8)$
$14=-p+8$
-8 -8
$\underline{6}=-\mathrm{p}$
-1 -1
$-6=p$

## LIST OF ALGEBRAIC PROPERTIES

| Property ( $a, b$ and $c$ are real numbers) | Example |
| :--- | :---: |
| Identity Property of Addition <br> Any number when added to 0 is itself. | $c+0=c$ |
| Identity Property of Multiplication <br> Any number multiplied by 1 is itself. | $b \cdot 1=b$ |
| Multiplicative Property of Zero <br> Any number multiplied by zero is zero. | $a \cdot 0=0$ |
| Commutative Property of Addition or Multiplication <br> The order in which you add or multiply does not change the sum or product. | $a+b=b+a$ <br> $a b=b a$ |
| Associative Property of Addition or Multiplication <br> The order in which you group numbers does not change the sum or product. | $(a+b)+c=a+(b+c)$ <br> $(a b) c=a(b c)$ |
| Symmetric Property <br> If one number is equal to a second number, then the second number is equal to the first. | If $a=b$ then $b=a$ |
| Substitution Property <br> If $a=b$ then $a$ can be substituted into an equation for $b$. | If $a=b$ and $b+c=6$ <br> then $a+c=6$. |
| Distributive Property <br> The product of a number and a sum or difference is the same as the sum or difference of <br> the products of the number and each element of the sum or difference. | $a(b+c)=a b+a c$ <br> $a(b-c)=a b-a c$ |
| Additive Inverse <br> The sum of a number and its inverse is zero. | $a+(-a)=0$ |
| Multiplicative Inverse <br> The product of a number and its inverse is one. | $b \cdot \frac{1}{b}=1$ |
| Addition Property of Equality <br> Adding the same number to each side of an equation produces an equivalent equation. | If $a=b$ then $a+c=b+c$ |
| Subtraction Property of Equality <br> Subtracting the same number from each side of an equation produces an equivalent <br> equation. | If $a=b$ then <br> $a-c=b-c$ |
| Multiplication Property of Equality <br> Multiplying each side of the equation by the same number produces an equivalent <br> equation. | If $a=b$ then <br> $a c=b c$ |
| Division Property of Equality <br> Dividing each side of the equation by the same number produces an equivalent equation. | If $a=b$ then $\frac{a}{c}=\frac{b}{c}$ |

Math 1 Review Sample Questions:

1. $-8=-(x+4)$
2. $-18-6 k=6(1+3 k)$
3. $5 n+34=-2(1-7 n)$
4. $2(4 x-3)-8=4+2 x$
5. $3 n-5=-8(6+5 n)$
6. $-(1+7 x)-6(-7-x)=36$

## EVALUATING AN EXPRESSION BY PLUGING IN VALUE

To evaluate an algebraic expression, plug in the given values for the unknowns and calculate according to PEMDAS.

Example: Find the value of $x^{2}+5 x-6$ when $x=-2$
Solution: Plug in -2 for $x$ and solve
$(-2)^{2}+5(-2)-6=4-10-6=-12$
Sample Questions:
7. Find the value of $5 a^{2}-6 a+12$ when $a=-5$
8. If $n=10$, then which of the following represents 552 ?
a. $\quad 5 n+2$
b. $\quad 5 n^{2}+2$
c. $\quad 5 n^{2}+5 n+2$
d. $\quad 5 n^{3}+5 n+2$
e. $\quad 5 n^{4}+5 n+2$
9. When $n=1 / 4$, what is the value of $\frac{2 n-5}{n}$
10. What is the value of the expression $10(100 x-10,000)+100$ when $x=250$ ?

## MULTIPLE PLUG INS

If there are more than one variable, just plug in each of the numbers into the corresponding variables.

## Sample Questions:

11. If $x=-4, y=3$ and $z=8$, what is the value of $2 x^{2}+3 y-z$ ?
12. What is the value of the expression $(x-y)^{2}$ when $x=5$ and $y=-1$ ?

## SOLVING A LINEAR EQUATIONS (SOLVE FOR X)

To solve an equation, isolate the variable. As long as you do the same thing to both sides of the equation, the equation is still balanced. To solve

Example: $5 x-12=-2 x+9$
Solution: First get all the $x$ terms on one side by adding $2 x$ to both sides: $7 x-12=9$. Then add 12 to both sides: $7 \mathrm{x}=21$, then divide both sides by 7 to get: $\mathrm{x}=3$.

Sample Questions:
13. For what value of $a$ is the equation $3(a+5)-a=23$ true?
14. If $4(x-2)+5 x=3(x+3)-11$, then $x=$ ?
15. If $x+\frac{2}{3}=\frac{8}{21}$, then $x=$ ?
16. When $\frac{1}{3} k+\frac{1}{4} k=1$, what is the value of $k$ ?
17. When $\frac{1}{2}(8 x+4)+\frac{2}{3}(3+6 x)=6$, what is the value of $x$ ?

## PLUGGING IN THE ANSWERS (PITA)

"Plugging $\ln$ " is a great strategy when there are variables in the question or the answers. Use Plugging In

- when there are variables in the answer choices
- when solving word problems or plug-and-chug questions
- for questions of any difficulty level

How about when there aren't? Does that mean we have go back to algebra? Of course not! On most problems on the ACT, there are a variety of ways to solve. Plugging in the answers are very helpful when:

- answer choices are numbers in ascending or descending order
- the question asks for a specific amount. Questions will usually be "what?" or "how many?"
- you have the urge to do algebra even when there are no variables in the problem

Because many answer choices are listed in ascending order, it will be best to start with the middle choice. That way, if it's too high or too low, we'll be able to use process of elimination (POE) more efficiently.

Simple Example: Solve for $12=-4(6 x-3)$
a. 0
b. 1
c. 2
d. 3
e. 4

Solution: Since the list is in ascending order, start in the middle and plug in 2 for x . When you do you get 6*2 which is $12-3$ which is 9 times -4 which is -36 . That is not it so try $1.6 * 1$ is $6-3$ is 3 times $-4=-12$. That is still too low so try $0.6^{*} 0$ is $0-3$ is -3 times -4 is 12 . That is the answer!

Note: This is a simple problem, so it is probably easier just to solve for x . However, this is a great skill to use when it is NOT simple or you can't figure it out another way.

More Complicated Example: In a piggy bank, there are pennies, nickels, dimes, and quarters that total \$2.17 in value. If there are 3 times as many pennies as there are dimes, 1 more dime than nickels, and 2 more quarter than dimes, then how many pennies are in the piggy bank?
a. 12
b. 15
c. 18
d. 21
e. 24

## Solution:

1. Know the question. How many pennies are in the bank?
2. Let the answers help. There are no variables, but the very specific question coupled with the numerical answers in ascending order gives a pretty good indication we can PITA.
3. Break the problem into bite-sized pieces. Make sure you take your time with this problem, because you'll need to multiply the number of each coin by its monetary value. In other words, don't forget that 1 nickel will count for 5 cents, 1 dime will count for 10 cents, and I quarter will count for 25 cents. Let's set up some columns to keep our work organized and begin with choice (C).

Since ACT has already given us the answers, we will plug those answers in and work backwards. Each of the answers listed gives a possible value for the number of pennies. Using the information in the problem, we can work backwards from that number of pennies to find the number of nickels, dimes, and quarters. When the values for the number of coins adds up to $\$ 2.17$, we know we're done.

If we begin with the assumption that there are 18 pennies, then there must be 6 dimes ( 3 times as many pennies as there are dimes). 6 dimes means 5 nickels ( 1 more dime than nickels) and 8 quarters ( 2 more quarters than dimes).

Now multiply the number of coins by the monetary value of each and see if they total \$2.17.

$$
\begin{array}{lllll}
\text { Pennies }(\$ P) & \text { Dimes }(\$ 0) & \text { Nickels }(\$ N) & \text { Quarters }(\$ Q) & \text { Total }=\$ 2.17 \\
\text { c. } 18(\$ 0.18) & 6(\$ 0.60) & 5(\$ 0.25) & 8(\$ 2.00) & \text { Total }=\$ 3.03(\mathrm{NO})
\end{array}
$$

That's too high, so not only is choice (C) incorrect, but also choices (D) and (E). Cross them off and try choice (B).

| Pennies $(\$ P)$ | Dimes $(\$ \mathrm{D})$ | Nickels $(\$ \mathrm{~N})$ | Quarters $(\$ \mathrm{Q})$ | Total $=\$ 2.17$ |
| :--- | :--- | :--- | :--- | :--- |
| a.15 $(\$ 0.15)$ | $5(\$ 0.50)$ | $4(\$ 0.20)$ | $7(\$ 1.75)$ | Total $=\$ 2.60($ NO $)$ |
| a. $12(\$ 0.12)$ | $4(\$ 0.40)$ | $3(\$ 0.15)$ | $6(\$ 1.50)$ | Total $=\$ 2.17($ YES $)$ |

Notes: Plugging In and PITA are not the only ways to solve these problems. The "best" way is the easiest way you can answer the problems. When you solve in a more complex way, there are a lot more opportunities to make careless errors.

Plugging in is not just for algebra. Remember what the main requirements are for a Plugging In problem. You need variables in the answer choices or question: that's it. It doesn't say anywhere that the problem needs to be a pure algebra problem. What is a pure algebra problem anyway? Don't forget: Part of what makes this test so hard is that ACT piles concept on cop of concept in its problems. In other words, geometry problems often are algebra problems.

## Sample Questions:

18. Which of the following is NOT a solution of $(x-5)(x-3)(x+3)(x+9)=0$ ?
a. 5
b. 3
c. -3
d. -5
e. -9
19. What is the value of $p$ in the following equation: $-11+10(p+10)=4-5(2 p+11)$ ?
a. -10
b. -7
c. -5
d. 0
e. 5
20. $-12(x-12)=-9(1+7 x)$
a. -3
b. -1
c. 1
d. 3
e. 5

More Sample Questions:
21. $-\frac{4}{7}=\frac{2 x+5}{x-8}$
22. $\mathrm{h}+\mathrm{j}(\mathrm{j}-\mathrm{h})$; use $\mathrm{h}=2$ and $\mathrm{j}=6$
23. $-10(-9 x+9)-8 x=9+3 x-4(2 x+3)$
24. $\frac{2}{3}=-\frac{3}{5} t$
25. $(b-3)+a b+(b+4)$; use $a=6$ and $b=1$
26. $9(7 x+4)-3(2 x-5)=3 x-4(x+2)+x(3-5)-1$
27. $\frac{3}{4}(2 x+1)=2$
28. $y(x-(9-4 y)) ;$ use $x=4$ and $y=2$
29. $-2(2 w+4)-3(2 w-2)=10 w+9-11-w$
30. $\frac{1}{3}+2 m=m-\frac{3}{2}$
31. $-6(m-4)=-10 m+3(8+8 m)$
32. $j(h-9)+2$; use $h=9$ and $j=8$
33. $15(2 x)+15-3=16+30 x$
34. $-8(-5 x-6)+7(1+9 x)=53+11 x(9)$
35. If $8 y=3 x-11$, then $x=$
a. $(88 / 3) y$
b. $\quad(8 / 3) y+11$
c. $\quad(8 / 3) y-11$
d. $\quad(8 y-11) / 3$
e. $\quad(8 y+11) / 3$

## Answer Key

1. 4
2. -1
3. 4
4. 3
5. -1
6. 5
7. 167
8. $5 n^{2}+5 n+2$ or C
9. -18
10. 150,100
11. 33
12. 36
13.4
13. 1
14. $-\frac{2}{7}$
15. $12 / 7$
16. 1/4
17. -5 or D
18. -7
19. -3
20. $-1 / 6$
21. 26
22. 1
23. $-10 / 9$ or $-11 / 9$
24. 9
25. -1
26. 5/6
28.6
27. 0
28. $-11 / 6$ or $-15 / 6$
31.0
29. 2
30. No solution
31. $-1 / 2$
32. E
