Name:__________________________________ Date:________________________

Math 2 Proportion & Probability Part 1
Percent, Ratio, Proportion, Rate, Average, Mean/Median/Mode, Combinations & Probability

PRE ALGEBRA REVIEW
DEFINITIONS

Ratio: A comparing two things **Proportions:** Two equivalent ratios **Rate:** Comparing two units of measure

IDENTIFYING THE PARTS AND THE WHOLE

The key to solving most fraction and percent word problems is to identify the part and the whole. Usually, you'll find the **part** associated with the verb ***is/are*** and the **whole** associated with the word ***of.*** In the sentence, "Half of the boys are blonds," the whole is the boys (*"of* the boys"), and the part is the blonds (*"are* blonds"). Sometimes, the problem will give you just the parts: "The ratio of boys to girls is 2 to 3." In this case, we have a part:part relationship. To find the ratio of boys to the total, you would add the parts: 2 + 3 = 5, so the ratio of boys to total (part:whole) is 2 to 5.

Percentages, probabilities, and ratios all come from the same basic concept: parts to wholes. Let's say you're taking batting practice. You are thrown 100 pitches and you hit 20 of them. First, what percent of the pitches did you hit? Well, that's easy on this one because we're dealing with 100. But you can always find percentages with this simple part-to-whole formula:

$$\frac{\text{part}}{\text{whole}} \text{ x } 100\%$$

Once we put the numbers in, we'll see this: $\frac{20}{100}$ x 100% = 0.2 x 100% = 20%. If you hit 20 of the 100 pitches, you hit 20% of them.

Next, what is the probability that you were to hit any given pitch? Remember, this is just a matter of parts to wholes, so we can find this probability as follows: $\frac{\text{part}}{\text{whole}} = \frac{\text{hits}}{\text{pitches}} = \frac{20}{100} = 0.2$. In other words, there's a 0.2, or 1/5 probability you hit any given pitch during batting practice.

Ratios are a little different. Usually these will ask for the relationship of some part to some other part. If we're still using our batting practice statistics, we might want to know something like, What's the ratio of the pitches you hit to the pitches you missed? Even though we're not dealing with the whole, we'll find the ratio the same way, but instead of $\frac{\text{part}}{\text{whole}}$, we'll use $\frac{\text{part}}{\text{part}} = \frac{\text{hits}}{\text{misses}} = \frac{20}{80} = \frac{1}{4}$. The ratio of hits to misses is ¼, or 1 to 4, or 1:4. If it feels like we just did the same thing three times, it was supposed to. As we've seen a few times already, just because things have different names doesn't mean that they are unrelated.

PERCENT FORMULA

In percent questions, whether you need to find the part, the whole, or the percent, use the same formula:

Part = Percent x Whole

When you use the formula, be sure to convert the percent into decimal form:

Example: What is 12% of 25?
Solution: Part = 0.12 x 25

Example: 15 is 3% of what number?
Solution: 15 = .03 x Whole

Example: 45 is what percent of 9?
Solution: 45 = Percent x 9

ANOTHER WAY TO DEAL WITH PERCENTS
A percentage is a fraction in which the denominator equals 100. In literal terms, the word percent means "divided by 100".

	Math Equivalent
Percent	/100
Of	Multiplication (x)
What	Variable (y, z)
Is, are, were	=
What percent	y/100

PERCENTAGE SHORTCUTS
In the last problem, we could have saved a little time if we had realized that 1/5 = 20 percent. Therefore, 4/5 would be 4 x 20 percent or 80 percent. Below are some fractions and decimals whose percent equivalents you should know.

1/5 = 0.2 = 20%
¼ = 0.25 = 25%
1/3 = $0.\overline{33}$ = 33 1/3%
½ = 0.5 = 50%

You can use a combination of techniques do even very complicated percentages by breaking them own into easy-to-find chunks.

20% of 500: 10% of 500 = 50, so 20% is twice 50, or 100.
30% of 70: 10% of 70 = 7, so 30% is three times 7, or 21.

Example: 32% of 400:
Solution: 10% of 400 = 40, so 30% is three times 40, or 120. And 1% of 400 = 4, so 2% is two times 4, or 8. Therefore, 32 percent of 400 = 120 + 8 = 128.

Sample Questions:
1. What is 270% of 60
2. 73% of what is 156.4?
3. 9 is what percent of 84?
4. 120% of 118 is what?
5. What is 1/5 of 250?
6. 18 is what fraction of 360?
7. 2/3 of 56 is what?
8. 45 is 4/5 of what?

PERCENT INCREASE AND DECREASE

To increase a number by a percent, **add the percent to 100%,** convert to a decimal, and multiply. To increase 40 by 25%, add 25% to 100%, convert 125% to 1.25, and multiply by 40. 1.25 x 40 = 50. To decrease, just subtract the percent from 100%, convert to a decimal, and multiply.

Example: John took his date to a restaurant and the bill was $32.54. The waiter was very good so John wants to tip 20%. What is the total amount John will pay for dinner?
Solution: John will pay 100% + 20% which is 120%. As a decimal, that is 1.2. Multiply 1.2 by $32.54 to get $35.05 as the total John will pay.

Example: Sam wants to buy a bike. The original price is $399.99, but it is on sale for 15% off. Excluding tax, what will Sam pay for the bike?
Solution: The sale is 100% - 15% which is 85%. As a decimal, that is 0.85. Multiply 0.85 by $399.99 to get $339.99 as the new price Sam would pay.
Sample Questions:

9. To keep up with rising expenses, a motel manager needs to raise the $40.00 room rate by 22%. What will be the new rate?

10. Jorge's current hourly wage for working at Denti Smiles is $12.00. Jorge was told that at the beginning of next month, his new hourly wage will be an increase of 6% of his current hourly wage. What will be Jorge's new hourly wage?

FINDING THE ORIGINAL WHOLE

To find the **original whole before a percent increase or decrease,** set up an equation with a variable in place of the original number. Say you have a 15% increase over an unknown original amount, say *x*. You would follow the same steps as always: 100% plus 15% is 115%, which is 1.15 when converted to a decimal. Then multiply to the number, which in this case is *x*, and you get 1.15x, then set that equal to the "new" amount.

Example: After a 5% increase, the population was 59,346. What was the population *before* the increase?
Solution: $1.05x = 59{,}346$
$x = 59{,}346/1.05 = 56{,}520$

Sample Questions:

11. Johnny is trying to figure out his original wage from work. 20% of Johnny's pay check is automatically taken out to pay taxes. The amount of money Johnny receives is $45,500. How much is Johnny's wage BEFORE taxes?

12. Allie ate 30% of all the candy. If the Allie ate 45 pieces, how much candy was there before she started eating it?

SETTING UP A RATIO

To find a ratio, put the number associated with the word ***of*** **on top** (as the numerator) and the quantity associated with the word *to* **on the bottom** (as the denominator), and reduce. The ratio of 20 oranges to 12 apples is $\frac{20}{12}$, which reduces to $\frac{5}{3}$. Be sure to keep the parts in the same order-so if a second ratio of oranges to apples was given, the oranges should be on top and the apples on the bottom.

Example: If the ratio of 2x to 5y is $\frac{1}{20}$, what is the ratio of x to y?

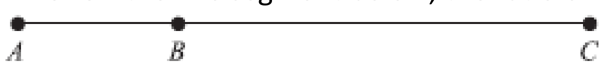
Solution: The difficulty of this problem is all in the setup. Just remember that you're comparing parts to wholes.

$\frac{2x}{5y} = \frac{1}{20}$. To isolate on the left side of chis equation, let's multiply both sides by 5/2.

$\frac{5}{2} x \frac{2x}{5y} = \frac{1}{20} x \frac{5}{2}$ $\frac{x}{y} = \frac{5}{40}$ 5/40 reduces to 1/8

Sample Questions:

13. On the line segment below, the ratio of lengths AB to BC is 1:5. What is the ratio of AC to AB?



a. 1:3
b. 1:7
c. 1:6
d. 6:1
e. Cannot be determined from the given information

14. In a shipment of cell phones, 1/25 of the cell phones are defective. What is the ratio of non-defective to total cell phones?

SIMPLE PROPORTION

Solving proportions is simply a matter of stating the ratios as fractions, setting the two fractions equal to each other, cross-multiplying, and solving the resulting equation.

Example: Find the unknown value in the proportion: 2 : *x* = 3 : 9.

Solution: First, convert the colon-based odds-notation ratios to fractional form: $\frac{2}{x} = \frac{3}{9}$

Then solve the proportion: $\frac{2}{x} = \frac{3}{9}$ 9(2) = *x*(3) 18 = 3*x* 6 = *x*

Example: Bountiful Baskets includes 3 oranges and 4 apples in every basket. How many oranges would there be in 4 baskets?

Solution: Set up ratios with the number of oranges in the numerator and the number of baskets in the denominator. For example, keep the original (known) numbers together in the same ratio and the new (unknown) numbers together in a second ratio and keep oranges on top and baskets on bottom.

$\frac{3\ oranges}{1\ basket} = \frac{x\ oranges}{4\ baskets}$ x = (3 * 4)/1 = 12

A second way to set up the proportion would be:

$\frac{x\ oranges}{3\ oranges} = \frac{4\ baskets}{1\ basket}$ x = (4*3)/1 = 12

Notice that the numerators contains the numbers from the new (unknown) situation and the denominators contain numbers from the original (known) situation AND the oranges are in the first ratio and the baskets are in the second ratio. There are actually 4 ways to correctly set up the proportions. The other two ways are:

$\frac{3\ oranges}{x\ oranges} = \frac{1\ basket}{4\ baskets}$ and $\frac{1\ basket}{3\ oranges} = \frac{4\ baskets}{x\ oranges}$

Proportions are usually set with x in the numerator since they are usually easier to solve, however since all proportions are correct, the answer will be the same.

RATE/PROPORTION

A rate is any "something per something"-days per week, miles per hour, dollars per gallon, etc. Pay close attention to the units of measurement, since often the rate is given in one measurement in the question and a different measurement in the answer choices. This means you need to convert the rate to the other measurement before you solve the question. This unit conversion can be done by setting up a proportion: Setting two equivalent rates equal to each other and solve for the unknown.

Example: If snow is falling at the rate of one *foot* every four hours, how many *inches* of snow will fall in seven hours?

Solution:

$\frac{1\ foot}{4\ hours} = \frac{x\ inches}{7\ hours}$ $\frac{12\ inches}{4\ hours} = \frac{x\ inches}{7\ hours}$ 4x = 12 x 7 x = 21

Example: How many feet in 7 yards?

Solution: Remember that there are 3 feet per 1 yard. Set up the proportion as

$\frac{x\ feet}{7\ yards} = \frac{3\ feet}{1\ yard}$ x = (3*7)/1 = 21 feet

or

$\frac{7\ yards}{1\ yards} = \frac{x\ feet}{3\ feet}$ or $\frac{1\ yards}{7\ yards} = \frac{3\ feet}{x\ feet}$ or $\frac{7\ yards}{x\ feet} = \frac{1\ yard}{3\ feet}$

Since all proportions are set up correctly, the answer will be 21 feet every time.

Sample Questions:

15. An oil refinery produces gasoline from crude oil. For every 10,000 barrels of crude oil supplied, the refinery can produce 5,500 barrels of gasoline. How many barrels of gasoline can be produced from 3,500 barrels of crude oil?

16. As a salesperson, your commission is directly proportional to the dollar amount of sales you make. If your sales are $800, your commission is $112. How much commission would you earn if you had $1,400 in sales?

17. There are 16 ounces in one pound. If 4.1 pounds of fish cost $9.95, what is the cost per ounce, to the nearest cent?

18. Jordan went for a 5-mile jog on Wednesday that took him 75 minutes. If on Tuesday Jordan jogs at the same rate of speed, how far will he jog in 45 minutes?

19. Jose took a trip to Mexico. Upon leaving he decided to convert all his Pesos back into dollars. How many dollars did he receive if he exchanged 42.7 Pesos at a rate of $5.30 = 11.1 Pesos?

MATH 1 LEVEL
PERCENT

Many percent problems in real life and on the ACT require multiple steps and multiple equations. Break the questions into bite-sized pieces and carefully solve.

Example: When 15% of 40 is added to 5% of 260, the resulting number is:
Solution: First, 15% of 40. Remember, % translates ÷ 100 and "of" translates to multiplication. Therefore, we can rewrite 15% of 40 as $\frac{15}{100}$ x 40. Put this expression in your calculator to find that $\frac{15}{100}$ x 40 = 6.
Now, 5% of 260. Use the same translations to find $\frac{5}{100}$ x 260 = 13. We've done the tough part, so let's substitute what we've found back into the problem: When 6 is added to 13, the resulting number is: 6 + 13 = 19.

Sample Questions:

20. If 20% of a number is 125, what is 28% of the number?

21. In the process of milling grain, 3% of the original is lost because of spillage, and another 5% of the original is lost because of mildew. If the mill starts out with 490 tons of grain, how much (in tons) remains to be sold after milling?

FINDING A PERCENT OF A PERCENT

When you learned how to translate simple English statements into mathematical expressions, you learned that "of" can indicate "times". This frequently comes up when using percentages. To take a percent of a percent of a number, use the same procedure and multiply the percent times the second percent times the number.

Example: Find 15% of 15% of 2000
Solution: Covert percentage to decimal form and multiply: 0.15 x 0.15 x 2000 = 45

Sample Questions:

22. What is 1/4 of 20% of $50,000?

23. At Paradigm, students must take both a written exam and an oral exam. In the past, 90% of the students passed the written exam and 75% of those who passed the written exam also passed the oral exam. Based on these figures, about how many students in a random group of 350 students would you expect to pass both exams?

PERCENT INCREASE AND/OR DECREASE

Percent is often used in real life money situations. Stores offer sales that are a certain % discount which means that the price paid is the percent of the original price subtracted from the original amount. There are also markups, taxes, etc. that are a percent of the original price added to the original price. In many realistic situations, there will be several increases, decreases or a combination of each. For example: a mark-up and a discount or a mark-up (like a tip) and a tax, etc.

Example: At a restaurant, diners enjoy an "early bird" discount of 10% off their bills. If a diner orders a meal regularly priced at \$18 and leaves a tip of 15% of the discounted meal (no tax), how much does he pay in total?

Solution:

10% = discount $\frac{10}{100} = \frac{\text{discount}}{\$18}$ Solving, the discount = \$1.80

The price of the discounted meal, then, is \$18 - \$1.80 = \$16.20. Let's find the tip the same way.

15% =tip $\frac{15}{100} = \frac{\text{tip}}{\$16.20}$ Tip = \$2.43

The price of the discounted meal plus tip, therefore, is \$16.20 + \$2.43 = \$18.63.

Sample Questions:

24. The original price of a pair of shoes is \$39.99. They are on sale 40% off and the tax rate is 6%. What is the final price?

25. Cost of dinner is \$29.95. You want to pay 20% tip and tax rate is 2%. What is the total cost of dinner?

RATE/PROPORTION

Rate and proportion can be seen in many different types of situations and may seem complex. However, just break the problems into smaller pieces and you will find that you have several simple proportions that may be solved consecutively.

Example: If 3.7 inches of rain fall on Vancouver during the first 4 days of November, and the rain continues to fall at this pace for the rest of the month, approximately how many feet of rain will fall during December? (Note: The month of December has 31 days.)

Solution: The ratio given in the problem is 3.7 inches every four days, or in math terms, $\frac{3.7\ in}{4\ days}$ The ratio will remain the same no matter how many days there are, so let's use this ratio to find how many inches of rain will fall in December.

$\frac{3.7\ in}{4\ days} = \frac{?\ in}{31\ days}$. Rearrange the terms: ? in = $\frac{(3.7\ in)(31 days)}{4\ days}$ = 28.675 in. Since the ratio of inches is 12 inches to 1 feet: $\frac{12\ in}{1\ ft}$. So, $\frac{12\ in}{1\ ft} = \frac{28.675\ in}{?ft}$. Rearrange the equation to find: ? in = $\frac{28.675\ in\ (1\ ft)}{1\ ft}$ = 2.4 ft

Sample Questions:

26. Which is the best buy? 6 shirts for \$25.50, 4 shirts for \$18.00 or 5 shirts for \$21

27. Lauren took 12 hours to read a 360 page book. At this rate, how long will it take her to read a 400 page book?

AVERAGE FORMULA

To find the average of a set of numbers, **add them up and divide by the number of numbers.**

Average $= \frac{\text{Sum of the terms}}{\text{Number of the terms}}$

Example: Find the average of the five numbers 12, 15, 23, 40, and 40.
Solution: First add them: 12 + 15 + 23 + 40 + 40 = 130. Then divide the sum by 5: 130 ÷ 5 = 26.

The average formula works for more complicated numbers: fractions, percentages, complicated numbers with variables, etc. To solve the more complicated problems, follow sound mathematical practices to get the correct answer.

Example: What is the average of 1/20 and 1/30?
Solution: To get the average, you must add the fractions and divide by two. Don't just average the denominators!!!

Average $= \frac{sum}{number\ of\ terms} = \frac{\frac{1}{20}+\frac{1}{30}}{2} = \frac{\frac{3}{60}+\frac{2}{60}}{2} = \frac{\frac{5}{60}}{2} = \frac{\frac{1}{12}}{2} = \frac{1}{12} \times \frac{1}{2} = \frac{1}{24}$

If you averaged the denominators you would have gotten 1/25, but the answer is 1/24. Don't be fooled!

Example: What is the average of the expressions 2x + 5, 5x – 6, -4x + 2 ?
Solution: Don't let the algebraic terms confuse you, simply add them and divide by 3.

Average $= \frac{sum}{number\ of\ terms} = \frac{(2x+5)+(5x-6)+(-4x+2)}{3} = \frac{(3x+1)}{3} = x + \frac{1}{3}$

Sample Questions:

28. What is the average of 2/3 and 4/5?

29. What is the average of the expressions x + 4, 2x – 5, 2x + 1?

30. As part of a school report on the cost of milk, Sammy wants to find the average cost of milk from local stores. She surveys 4 stores and finds the cost per gallon of whole milk from the 4 stores to be $2.38, $2.50, $2.79, $2.46, respectively. Using this data, what is the average cost of purchasing one gallon of whole milk from these 4 gas stores?

31. Timothy's average score on the first 4 tests was 76. On the next 5 tests his average score was 85. What was his average score on all 9 tests?

32. Tracy mowed lawns for 2 hours and earned $7.40 per hour. Then she washed windows for 3 hours and earned $6.50 per hour. What were Tracy's average earnings per hour for all 5 hours?

33. After taking 3 quizzes, your average is 72 out of 100. What must your average be on the next two quizzes so that on 5 quizzes you increase your average to 77?

34. If the average of 8,11,25, and p is 15, find p.

35. If a = 3b = 6c, what is the average of a, b and c in terms of a?

WEIGHTED AVERAGE

Another type of average problem involves the weighted average - which is the average of two or more terms that do not all have the same number of members. To find the weighted term, multiply each term by its weighting factor, which is the number of times each term occurs. Use weighted average to calculate GPA or figure out mixtures.

The formula for weighted average is:

$$Weighted\ Average = \frac{Sum\ of\ Weighted\ Terms}{Total\ Number\ of\ Terms}$$

Example: A class of 25 students took a science test. 10 students had an average score of 80. The other students had an average score of 60. What is the average score of the whole class?

Solution:
$$Weighted\ Average = \frac{Sum\ of\ Weighted\ Terms}{Total\ Number\ of\ Terms}$$
$$= \frac{1700}{25}$$
$$= 68$$

Be careful! You will get the wrong answer if you add the two average scores and divide the answer by two.

Example: First semester, Joanie got a 4 in Seminar (which counts for two class periods), 3 in chemistry, 3 in math, 4 in art, 4 in choir, 2 in novel writing and 2 in advanced fitness. What is Joanie's GPA?

Solution: Count the number of classes that Joanie received a 4 and multiply that together. Add that to the number of classes that Joanie received a 3 multiplied by 3. Add that by the number of classes that Joanie received a 2 and multiply that by 2. Then divide the total by 8 (the total number of classes Joanie took in first semester. $\frac{4(4) + 3(2) + 2(2)}{8} = 3.25$

Example: Emily mixed together 9 gal. of Brand A fruit drink and 8 gal. of Brand B fruit drink which contains 48% fruit juice. Find the percent of fruit juice in Brand A if the mixture contained 30% fruit juice.

Solution: Emily mixed 9 gal of A with 8 gal of B which makes 17 gal of total juice. We are trying to calculate the % fruit juice in A, so that is labeled X. Add the amount of fruit juice in A and B and set that equal to the amount of juice in the final solution. Amount fruit juice is calculated as per weighted average: number of gallons times the percent fruit juice.

9x + 8(0.48) = 17(0.30)

9x + 3.84 = 5.1

9x = 5.1 – 3.84 = 1.26

x = 1.26/9 = 0.14 = 14% There is 14% fruit juice in Brand A

Sample Questions:

36. Fifteen accounting majors have an average grade of 90. Seven marketing majors averaged 85, and ten finance majors averaged 93. What is the weighted mean for the 32 students?

37. 5 fl. oz. of a 2% alcohol solution was mixed with 11 fl. oz. of a 66% alcohol solution. Find the concentration of the new mixture.

A	4.0	C	2.0
A-	3.7	C-	1.7
B+	3.3	D+	1.3
B	3.0	D	1.0
B-	2.7	F	0.0
C+	2.3		

38. James must have a 3.0 GPA to be able to play on the team. He already has a B-, B-, C, B-, A, A, B-. What is the minimum grade he needs to get in his last class to be able to play on the team?

AVERAGE RATE: DISTANCE RATE PROBLEMS

Distance = Rate x Time

The first important formula to memorize is: D = R x T. This stands for Distance = Rate x Time. Think of it as the "DIRT" formula and writing it this way is an easy way to remember. It is perfectly acceptable to also think of it as Time = Distance / Rate or as Rate = Distance / Time. Usually the "Rate" is speed but it could be anything "per" anything. In a word problem, if you see the word "per" you know this is a question involving rates.

Average rate is its own special concept and you will notice that it is NOT an Average of the Speeds (which would be something like the Sum of the Speeds / the Number of Different Speeds or what we know as the Arithmetic Mean). Average Rate is completely different. The most common rate is speed-distance over time-and the most common question about average rates is average speed-total distance over total time.

Average Rate = Total Distance / Total Time

Example: Ariella drove 40 miles to see her cousin and at 20 mph. The trip took Ariella 2 hours. Then, Ariella drove from her cousin's house and drove another 30 miles to the store at a speed of 10mph. It took Ariella 3 hours to arrive at the store. What was Ariella's average speed for the trip?

Solution: Average Speed = Total Distance / Total Time.

Ariella traveled 40 miles + 30 miles, so her Total Distance was 70 miles. She drove for 2 hours + 3 hours, so her Total Time was 5 hours. 70/5 = 14.

Her Average Speed for the whole trip was 14 mph.

The Average Speed in this problem is 14 mph, which is different from the "Average of the Speeds." If we had just averaged the two speeds (10mph and 20mph), we would have gotten 15mph. Think of Average Speed as a weighted average. Because Ariella spent more time in the problem going 10mph than 20mph, so it makes sense that the Average Speed would be closer to 10mph. Be careful! The "Average of the Speeds" will often be a tempting wrong answer choice on the ACT!

Example: Tracey ran to the top of a steep hill at an average pace of 6 miles per hour. She took the exact same trail back down. To her relief, the descent was much faster; her average speed rose to 14 miles per hour. If the entire run took Tracey exactly one hour to complete and she did not make any stops, what is the length of trail in miles one way?

Solution: For the way up the hill, we know that D = 6mph x T.

For the way down the hill, we know that D = 14mph x T. Since we went know that the distance up the hill was the same as the distance down the hill, we can pick a number for D. Let's choose "84" since it is a multiple of both 6 and 14. If 84 = 6mph x T, then we know that T = 14 hours. If 84 = 14mph x T, then we know that T = 6 hours. Now we can use another formula, the Average Rate formula, to find the average speed for the WHOLE trip. Average Rate = Total Distance / Total Time Using our Picked Number of 84, we know that the Total Distance traveled would be 168 miles. The Total Time is 14 hours + 6 hours = 20 hours. So the Average Rate = 168 miles / 20 hours = 8.4 mph. It doesn't matter that Tracey didn't "really" go 168 miles, or that we know she didn't "really" go 20 hours. We Picked a Number just so that we could find the ratio of the Total Distance to the Total Time in order to calculate the Average Rate of the ENTIRE journey.

Now that we have found the Average Rate for the whole trip, we can plug it in to the "DIRT" formula to find the ACTUAL distance for the entire journey.

D = R x T

D = 8.4mph x 1 hour

We know that T = 1 hour because the problem told us so. Therefore, the actual distance for the entire trip was 8.4 miles. The problem asks how many miles the trail was one way. 8.4 / 2 = 4.2. The answer to the question is 4.2 miles.

Sample Questions:

39. Robert rides his scooter an average of 5 miles per hour, and Cindy rides her roller blades an average of 6 miles per hour. At these rates, how much longer does it take Runner A than Runner B to travel 3 miles?

40. A cargo plane flew to the maintenance facility and back. It took one hour less time to get there than it did to get back. The average speed on the trip there was 220 mph. The average speed on the way back was 200 mph. How many hours did the trip there take?

41. Kali left school and traveled toward her friend's house at an average speed of 40 km/h. Matt left one hour later and traveled in the opposite direction with an average speed of 50 km/h. Find the number of hours Matt needs to travel before they are 400 km apart.

42. A submarine left Hawaii two hours before an aircraft carrier. The vessels traveled in opposite directions. The aircraft carrier traveled at 25 mph for nine hours. After this time the vessels were 280 mi. apart. Find the submarine's speed.

AVERAGE RATE: WORK RATE PROBLEMS

To work out these problems, it is usually assumed that a work-agent (say a man) takes certain number of time units T (usually days or hours) to complete the work. So the work rate of the agent in one time unit (a day or an hour) is expressed as $\frac{1}{T}$th portion of the total amount of work. The reason why this work rate in terms of work portion per unit time is the most important concept in Time and Work problems is - it makes possible summing up of efforts of more than one type of work agents working together at different work rates over unit time. This is the core concept behind the deductions of Time and Work problem of any type.

Example: A job can be completed by 4 men in 24 days and 4 women in 12 days. In how many days would the 4 men and 4 women working together complete the work?

Solution: In case of 4 men and 4 women working independently completing a job in 24 days and 12 days respectively, if we are asked to find the number of days taken to complete the job by 4 men and 4 women working together, by the conventional approach, we derive the per day work rate of 1 man and 1 woman as, $\frac{1}{4x24} = \frac{1}{96}$ portion of work, and $\frac{1}{4x12} = \frac{1}{48}$ portion of work.

When these two teams work together for 1 day, we would now be able to sum up their efforts in one day as, $\frac{4}{96} + \frac{4}{48} = \frac{3}{24} = \frac{1}{8}$ portion of work.

We now arrive at the desired result using unitary method. The number of days that the two teams would take to complete the job working together would just be inverse of the per day work portion, that is, 8 number of days.

This approach seems to be a bit complex as it deals with inverses, but this is the usual method followed.

Sample Questions:

43. It takes Trevon ten hours to clean an attic. Cody can clean the same attic in seven hours. Find how long it would take them if they worked together.

44. Working alone, Carlos can oil the lanes in a bowling alley in five hours. Jenny can oil the same lanes in nine hours. If they worked together how long would it take them?

MEAN, MEDIAN, MODE, RANGE

Mean
The "mean" is the "average", where you add up all the numbers and then divide by the number of numbers.

Median
The "median" is the "middle" value in the list of numbers, like the median strip on the highway. To find the median, your numbers have to be listed in numerical order, so you may have to rewrite your list first.

Mode
The "mode" is the value that occurs most often--mOde, mOst. If no number is repeated, then there is no mode for the list.

Range
The "range" is just the difference between the largest and smallest values.

Sample Questions:

45. What is the difference between the mean and the median of the set {2, 5, 18, 22} ?

46. To increase the mean of 4 numbers by 2, by how much would the sum of the 4 numbers have to increase?

47. The weekly salaries of six employees at McDonalds are $140, $220, $90, $180, $140, $200. For these six salaries, find: (a) the mean (b) the median (c) the mode and (d) the range.

COUNTING THE POSSIBILITIES
The fundamental counting principle: If there are **m ways** one event can happen and ***n* ways** a second event can happen, then there are **m x *n* ways** for the two events to happen. For example, with 7 shirts and 5 pairs of pants to choose from, you can put together 7 x 5 = 35 different outfits. For more items, multiply all the ways together. For example, with 7 shirts, 5 pairs of pants, 8 ties and 2 pairs of shoes to choose from, you can put together 7 x 5 x 8 x 2 = 560 different outfits.

COMBINATIONS: THE SLOT METHOD
Combination problems ask you how many different ways a number of things could be chosen or combined.

- Figure out the number of slots you need to fill.
- Fill in those slots.
- Find the product.

Example: At the school cafeteria, students can choose from 3 different salads, 5 different main dishes, and 2 different desserts. If Isabel chooses one salad. one main dish, and one dessert for lunch, how many different lunches could she choose?

Solution: There are three slots to fill, one for each item: salad, main dish, dessert. And the number of possibilities for each is pretty clear. Set up the slots and take the produce as your answer.

3	x	5	x	2	= 30
Salad		Main		Dessert	

Sample Questions:

48. Reggie knows how to make 5 different entrees, 4 different side dishes, and 6 different desserts. How many distinct complete meals, each consisting of an entrée, a side dish, and a dessert, can Reggie make?

49. Chris owns 10 different dress shirts, 4 different pairs of pants, and 7 different ties. How many distinct outfits, each consisting of a shirt, a pair of pants, and a tie, can Justin make?

PROBABILITY

Probability is always the number of desired outcomes divided by the number of possible outcomes. It can be expressed as a fraction, or sometimes, as a decimal.

Probability = $\frac{Favorable\ outcomes}{Total\ possible\ outcomes}$

If you have 12 shirts in a drawer and 9 of them are white, the probability of picking a white shirt at random is $\frac{9}{12} = \frac{3}{4}$. This probability can also be expressed as 0.75 or 75%.

Example: Herbie's practice bag contains 4 blue racquetballs. one red racquetball, and 6 green racquetballs. If he chooses a ball at random, which of the following is closest to the probability that the ball will NOT be green?

a. 0.09
b. 0.27
c. 0.36
d. 0.45
e. 0.54

Solution:

Step 1: **Know the question.** Make sure you read carefully! We want the probability that the chosen ball will NOT be green.

Step 2: **Let the answers help.** Green balls account for slightly more than half the number of balls in the bag, so the likelihood that the ball will NOT be green could be slightly less than half. That eliminates choices (A), (B), and (E).

Step 3: **Break the problem into bite-sized pieces.** This is a pretty straightforward $\frac{\text{part}}{\text{whole}}$ problem. $\frac{\text{part}}{\text{whole}} = \frac{\text{NOT green}}{\text{all}} = \frac{5}{11}$.

The only slight difficulty is that the answers are not listed as fractions, but it's nothing a calculator can't help. Find 5 ÷·11 = 0.45, choice (D).

Sample Questions:

50. If a gumball is randomly chosen from a bag that contains exactly 7 yellow gumballs, 3 green gumballs, and 2 red gumballs, what is the probability that the gumball chosen is green?

51. There are 15 balls in a box: 7 balls are green, 3 are blue and 5 are white. Then 1 green and 1 blue balls are taken from the box and put away. What is the probability that a blue ball will NOT be selected at random from the box?

52. Seth has 4 plaid shirts and 5 solid-colored shirts hanging together in a closet. In his haste to get ready for work, he randomly grabs 1 of these 9 shirts. What is the probability that the shirt Seth grabs is plaid?

53. A partial deck of cards was found sitting out on a table. If the partial deck consists of 6 spades, 3 hearts, and 7 diamonds, what is the probability of randomly selecting a red card from this partial deck? (Note: diamonds and hearts are considered “red” while spades and clubs are considered “black”)?

Answer Key

1. 162
2. 214.2
3. 10.7%
4. 141.6
5. 50
6. 1/20
7. $37.\overline{3}$
8. 56.25
9. 48.8
10. 12.72
11. 56,875
12. 150
13. 6:1(d)
14. 24/25
15. 1,925
16. $196
17. $0.15
18. 3 miles
19. $1.38
20. 175
21. 450.8
22. 2,500
23. 236
24. $25.43
25. $36.56
26. n$25.50 / 6 shirts = $4.25 per shirt $18.00 / 4 shirts = $4.50 per shirt $21.00 / 5 shirts = $4.20 per shirt- this is the best buy
27. 360 pages / 12 hours = 30 pages per hour400 pages / 30 pages per hour = 13 1/3 hours = 13 hours and 20 minutes.
28. 11/15
29. 5x/3
30. $2.53
31. 81
32. $6.86
33. 84.5
34. 16
35. a/2
36. 90%
37. 46%
38. Need to have 3.2 or greater, so B+ to be able to play
39. 0.1 hr or 6 min longer
40. 10 hrs
41. 4 hrs
42. 5 mph
43. 4.12 hours
44. 3.21 hours
45. 0.25
46. 8
47.
    a) Mean = $161.\overline{6}$
    b) Median = 160
    c) Mode = 140
    d) Range = 130
48. 120
49. 280
50. ¼
51. 11/13
52. 4/9
53. 5/8