

Name: _____

Date: _____

Math 2 Coordinate Geometry Part 2

Lines & Systems of Equations

USING TWO POINTS TO FIND THE SLOPE - REVIEW

In mathematics, the slope of a line is often called m . We can find the slope if we have two points on the line. We'll call the first point (x_1, y_1) , and the second point (x_2, y_2) . To find the slope we make a fraction. Take the y value from the second point and subtract the y value from the first point for the numerator and take the x value from the second point and subtract the x value from the first point for the denominator.

$$\text{slope} = m = \frac{\text{change in } y}{\text{change in } x} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

The slope of the line that contains the points A (2,3) and B (0,-1) is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{(-1) - (3)}{(0) - (2)} = \frac{-4}{-2} = 2$$

The slope of the line that contains the points A (-2,3) and B (0,-1) is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{(-1) - (3)}{(0) - (-2)} = \frac{-4}{2} = -2$$

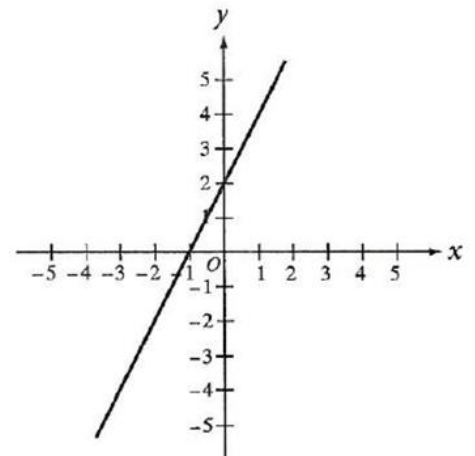
- In the coordinate plane a line runs through the points (2,3) and (4, 5). What is the slope of the line?
- In the Cartesian plane, a line runs through the points (5, 6) and (-2, -2). What is the slope of the line?

FINDING SLOPE AND Y-INTERCEPT FROM SLOPE-INTERCEPT FORM

Slope-intercept form is $y = mx + b$ where m is the slope and b is the y -intercept.

- What is the slope of the line $y = \frac{-1}{3}x - 7$?
- At what point does the line $y = \frac{2}{5}x + 3$ cross the y axis?
- Which of the following is an equation for the graph in the following standard (x,y) coordinate plane?

- $y = -2x + 1$
- $y = x + 1$
- $y = x + 2$
- $y = 2x - 1$
- $y = 2x + 2$



RE-WRITING EQUATIONS IN SLOPE-INTERCEPT FORM

To find the slope or y-intercept of a line that is not in slope-intercept form, re-write the equation in slope-intercept form. For example, to find the slope of the line $6y - 3x = 18$, re-write it so it looks like

$$y = mx + b$$

6. At which y-coordinate does the line described by the equation $6y - 3x = 18$ intersect the y-axis?
7. At which y-coordinate does the line described by the equation $y - 2x = 8$ intersect the y-axis?
8. What is the slope of the line represented by the equation $10y - 16x = 13$?
9. What is the slope of the line given by the equation $8 = 3y - 5x$?

CREATING AN EQUATION FOR A LINE FROM 2 POINTS

Finding an equation for a line when we are only given 2 points takes 2 steps. We know that $y = mx + b$, where m is the slope and b is the y-intercept. So in order to create the equation we need m and we need b .

Step 1: find m (slope)

We already know how to find the slope from 2 points.

$$\text{slope} = m = \frac{\text{change in } y}{\text{change in } x} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

Step 2: find b (y-intercept)

We're going to use the formula $y = mx + b$ to solve for b . Plug in the slope you found in step 1 for m , then choose one of the points that was given (it doesn't matter which one), plug in the x value for x in the formula and the y value in for y in the formula. Now the only variable that doesn't have a known value is b . Solve for b .

Example: What is the equation of the line that passes through the points A (2,3) and B (0,-1)?

Solution: Step 1: find m (slope)

$$\text{slope} = m = \frac{\text{change in } y}{\text{change in } x} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(-1) - (3)}{(0) - (2)} = \frac{-4}{-2} = 2$$

Step 2: find b (y-intercept)

$$y = mx + b \quad (\text{start with the generic slope-intercept formula})$$

$$3 = 2(2) + b \quad [\text{plugging in the slope (2) for } m \text{ and point (2,3) for } x \text{ and } y \text{ in the equation}]$$

$$3 = 4 + b \quad (\text{multiply})$$

$$-1 = b \quad (\text{subtract 4 from each side})$$

$$y = mx + b \quad (\text{now take the generic formula for a line and...})$$

$$y = 2x - 1 \quad (\text{plug in the values you found for } m \text{ and } b)$$

Sample Questions:

10. What is the equation of the line that passes through points A (-2,3) and B (0,-1)?

11. What is the equation of the line that passes through the points (1,0) and (5,3)?

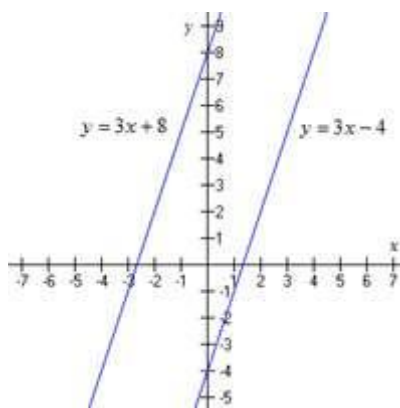
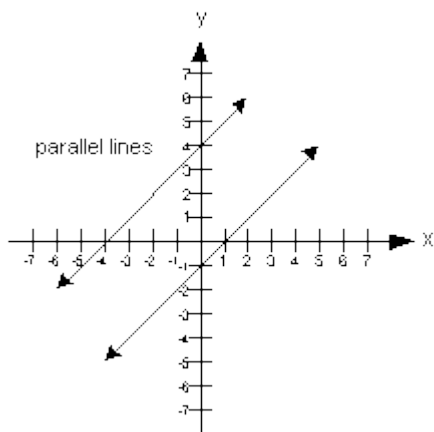
12. What is the equation of the line that passes through the points (3,0) and (1,2)?

13. In the coordinate plane a line runs through the points (2,3) and (4, 5). What is the equation of the line?

14. In the Cartesian plane, a line runs through the points (5, 6) and (-2, -2). What is the equation of the line?

PARALLEL LINES ON THE COORDINATE PLANE

Parallel lines always have the same slope. Therefore, if two lines have the same slope then they are parallel.



Sample Questions:

15. These 2 lines are parallel:

$$y = \frac{2}{3}x + 3$$

$$y = \frac{2}{3}x$$

True

False

16. These 2 lines are parallel:

$$y = \frac{-1}{2}x + 5$$

$$y = \frac{2}{3}x - 3$$

True

False

17. These 2 lines are parallel:

$$x + 2y = 3$$

$$2y = x - 7$$

True

False

18. These 2 lines are parallel:

$$3y - x = 6$$

$$3y = 2x + 9$$

True

False

19. Which of these is an equation of a line with a y-intercept of 3 and is parallel to the line $3y - x = 6$

A. $y = \frac{1}{3}x + 2$

B. $y = \frac{1}{3}x + 3$

C. $y = 3x + 2$

D. $y = 2x + 6$

E. $y = \frac{1}{2}x + 3$

20. Which of these is an equation of a line parallel to the line $2y = x - 7$ which goes through the point (0, 5)?

A. $y = 5x - 7$

B. $y = 2x + 5$

C. $y = \frac{1}{2}x - 7$

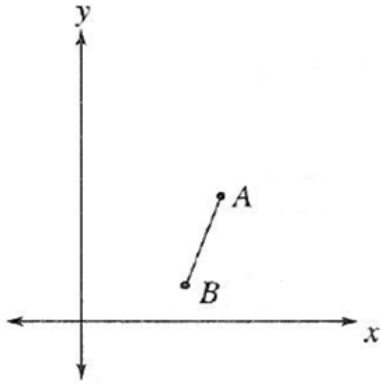
D. $y = \frac{1}{2}x + 5$

E. $y = \frac{-1}{2}x + 5$

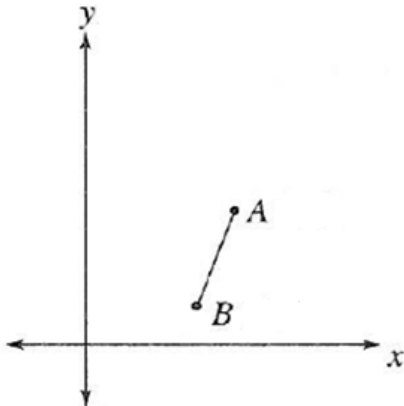
21. Write the equation of a line which is parallel to the line $y = \frac{2}{3}x + 3$ and passes through the point (0, -1)

22. The line that passes through the points (1, 1) and (2, 16) in the standard (x,y) coordinate plane is parallel to the line that passes through the points (-10, -5) and (a, 25). What is the value of a?

23. Side AB of parallelogram ABCD is shown in the figure below. If the coordinates of A are (9,8) and those of B are (6,2), what is the slope of side CD?



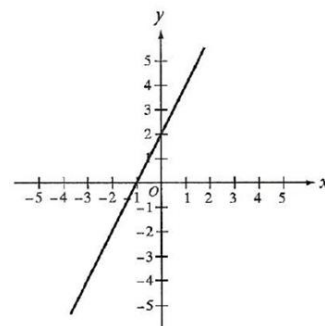
24. Side AB of parallelogram ABCD is shown in the figure below. If the coordinates of A are (7,6) and those of B are (5,1) then side CD could lie on which of the following lines?



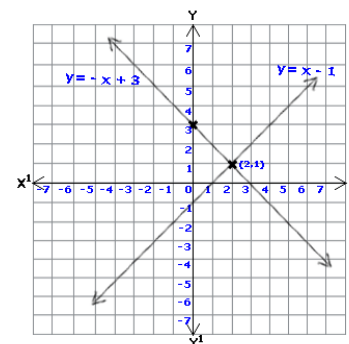
- A. $y = 3x + 1$
- B. $y = \frac{5}{2}x - 1$
- C. $y = \frac{2}{5}x + 7$
- D. $y = 2x - 6$
- E. Unable to determine from the information given

SYSTEMS OF EQUATIONS

When we have an equation with two variables, there is more than one possible answer. A line shows all the possible answers for a linear equation. For example, the line in the graph to the right shows all the possible answers to the equation $y = 2x + 2$.



However, if there are 2 linear equations there is only one possible answer; it is the point where the two lines meet. It is only at this point where the variables work for both equations. When it comes to systems of equations, X marks the spot. For example, in the graph below there are two equations, $y = x - 1$ and $y = -x + 3$. Although there are several possible answers for either line, there is only one spot (2, 1) that satisfies both equations.



$$\begin{aligned} y &= x - 1 && \text{(start with the equation)} \\ 1 &= 2 - 1 && \text{(plug in 2 for x and 1 for y)} \\ 1 &= 1 && \text{(it works)} \\ &&& \text{(now try it with the other equation)} \\ y &= -x + 3 && \text{(start with the equation)} \\ 1 &= -2 + 3 && \text{(plug in 2 for x and 1 for y)} \\ 1 &= 1 && \text{(it works)} \end{aligned}$$

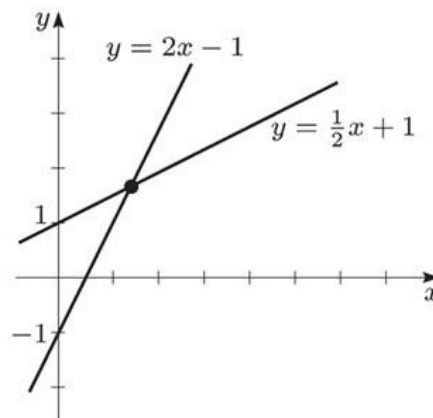
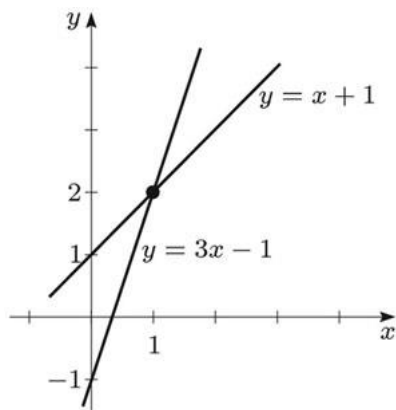
Let's try another point. In the equation $y = x - 1$, if x is 3 then y must be 2 so the point (3,2) works for that linear equation. Let's try that same point in the other equation.

$$\begin{aligned} y &= -x + 3 && \text{(start with the equation)} \\ 2 &= -3 + 3 && \text{(plug in the 3 for x and 2 for y)} \\ 2 &= 0 && \text{That doesn't work.} \end{aligned}$$

The ONLY point that works for BOTH equations is (2,1), the point of intersection of both lines.

SOLVING SYSTEMS OF EQUATIONS

When we have 2 linear equations, we can solve for the only x and y value that work for both equations. The x and y values will be the point of intersection for both lines. Sometimes we can find the point by graphing both lines and finding the point where the lines intersect, however, the lines don't always intersect at nice integer numbers and unless your graph is amazingly accurate, it is possible to make a mistake in finding the exact point of intersection. For example in the graph below and to the left, I'm confident that the point of intersection is (1,2) and I can plug in those points into both equations to verify that this point works in both equations. However, in the graph below and to the right, I'm not certain from looking at the graph of the point of intersection.



SOLVING SYSTEMS OF EQUATIONS BY SUBSTITUTION

Since graphing doesn't always work, there has to be another way to find the answer. One method is called substitution. We'll use the example from the graph above and to the right. We start with both equations. We're going to use part of one equation as a substitute for a variable in the other equation.

$y = 2x - 1$	Start with the original equation
$y = \frac{1}{2}x + 1$	Start with the original equation
$2x - 1 = \frac{1}{2}x + 1$	Since I know that $y = 2x - 1$, I can substitute $(2x - 1)$ for y in the other equation.
$2(2x - 1) = 2(\frac{1}{2}x + 1)$	Multiply both sides by 2 to get rid of that annoying fraction.
$4x - 2 = x + 2$	Distributive property and simplify
$3x - 2 = 2$	Subtract x from both sides. Now I only have an x on one side of the equation.
$3x = 4$	Add 2 to both sides.
$x = \frac{4}{3}$	Divide both sides by 3. Now I know that the x value is $\frac{4}{3}$, but I still don't know the y value
$y = \frac{1}{2}x + 1$	To find the y value I simply plug in the x value into one of the original equations. It doesn't matter which equation, since the goal is to find the one point (x,y) that works for both equations. However, when I look at my choices, one looks easier than the other, so I prefer to choose the easier one.
$y = 2x - 1$	I've chosen the equation, now I just have to plug in $\frac{4}{3}$ for x and solve for y .
$y = 2\left(\frac{4}{3}\right) - 1$	Plug in $\frac{4}{3}$ for x (because we already found that $x = \frac{4}{3}$)
$y = \frac{8}{3} - 1$	Simplify.
$y = \frac{8}{3} - \frac{3}{3}$	Re-write 1 as $\frac{3}{3}$ (common denominator)
$y = \frac{5}{3}$	Simplify.

Now I know both the x and the y value of the point of intersection for these two lines $\left(\frac{4}{3}, \frac{5}{3}\right)$. This is the only point that works for both equations.

Sample Questions:

25. Find the point of intersection for the lines $y = \frac{1}{2}x$ and $y = \frac{-1}{2}x + 4$.

26. Find the point of intersection for the lines $y = \frac{2}{3}x - 4$ and $y = -x + 1$.

27. Solve the system of linear equations

$$y = 3x - 1$$

$$y = -2x + 4$$

SOLVING SYSTEMS OF EQUATIONS BY ELIMINATION (ADDING AND SUBTRACTING)

There is another way to solve systems of equations. We'll solve the system of equations $2y + 3x = 5$ and $2y - x = 1$. We're looking for ways to eliminate one of the variables by adding or subtracting one entire equation from the other. I notice that both of the equations have $2y$ in them so if I subtract one equation from the other, the y value will disappear.

$$\begin{array}{ll} 2y + 3x = 5 & \text{Start with the original equation} \\ 2y - x = 1 & \text{Start with the original equation} \end{array}$$

$$\begin{array}{ll} 2y + 3x = 5 & \text{Start with the original equation} \\ -(2y - x = 1) & \text{multiply the other equation by -1} \end{array}$$

$$\begin{array}{ll} 2y + 3x = 5 & \text{Original equation} \\ \underline{-2y + x = -1} & \text{Use the distributive property} \\ 4x = 4 & \text{Add the two equations. } 2y - 2y = 0, 3x + x = 4x \text{ and } 5 - 1 = 4 \\ x = 1 & \text{Divide each side by 4. Now we know the } x \text{ value for the point of intersection.} \end{array}$$

$$\begin{array}{ll} 2y - x = 1 & \text{Start with the original equation (it doesn't matter which one)} \\ 2y - (1) = 1 & \text{Plug in 1 for } x \\ 2y = 2 & \text{Add 1 to both sides} \\ y = 1 & \text{Divide both sides by 2. Now we know the } y \text{ value for the point of intersection.} \end{array}$$

The solution to both equations is (1,1)

Sample Questions:

28. Find the point of intersection for the lines $2y - 3x = 0$ and $2y + x = 7$.

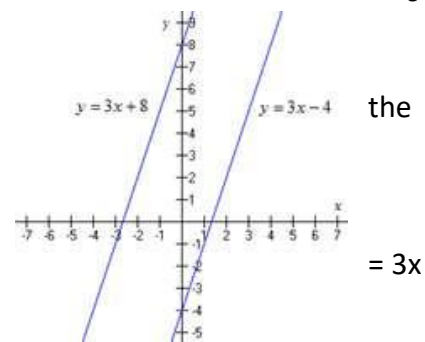
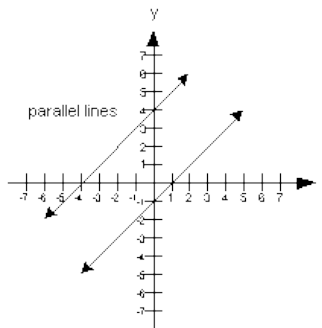
29. Find the point of intersection for the lines $3y + x = 1$ and $y - x = 5$.

30. Solve the system of linear equations

$$\begin{array}{l} 6y - 2x = 1 \\ 2y + 2x = 3 \end{array}$$

NO SOLUTIONS

Not every system of linear equations has a solution. Since the solution is the point where two lines intersect, if the lines never intersect, then there isn't a solution. This is the case whenever we have parallel lines. The example above and to the right graphs the equations $y = 3x + 8$ and $y = 3x - 4$. We'll try to solve by elimination and see what happens.



$y = 3x + 8$ Start with the original equation
 $-(y = 3x - 4)$ Multiply the other equation by -1 and use the distributive property
 $0 = 0 + 4$ Add the two equations ($y - y = 0$, $3x - 3x = 0$, and $8 - 4 = 4$)
 $0 = 4$ Simplify. This doesn't work. There aren't any solutions because the lines never intersect.

Sample Questions:

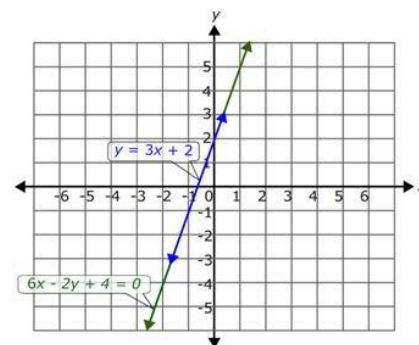
31. Which of the following systems of equations does NOT have a solution? (Hint: find the slope of the lines and see which are parallel)
- $x + 3y = 19$ and $3x + y = 6$
 - $x + 3y = 19$ and $x - 3y = 13$
 - $x - 3y = 19$ and $3x - y = 7$
 - $x - 3y = 19$ and $3x + y = 6$
 - $x + 3y = 6$ and $3x + 9y = 7$

INFINITE SOLUTIONS

There is a possibility of infinite solutions to a set of linear equations. There is only one way for this to happen and that is that the two lines are really the same line. For example, find the solution to the set of linear equations $y = 3x + 2$ and $6x - 2y + 4 = 0$. I can use either substitution or elimination to solve the equations. I choose substitution.

$y = 3x + 2$ Start with original equation
 $6x - 2y + 4 = 0$ Start with original equation
 $6x - 2(3x + 2) + 4 = 0$ Substitute $(3x + 2)$ for y since $3x + 2 = y$
 $6x - 6x - 4 + 4 = 0$ Distributive property
 $0 = 0$ Simplify. Although this answer is true, it isn't helpful.
 It is true for any value of x . So we just say "infinite solutions."

The reason we got this strange answer is that the two equations are really just the same equation written differently. Being able to write the same equation in a different way is very helpful in some situations. This is how we re-write equations in slope-intercept form. We rearrange the equation until it is in a form that we want. It is also helpful when solving systems of equations by elimination. We can multiply an equation by a 2 or a 3 or -1 or whatever, to get it into the form that helps us eliminate a variable. We'll explain more about that later.



Sample Questions:

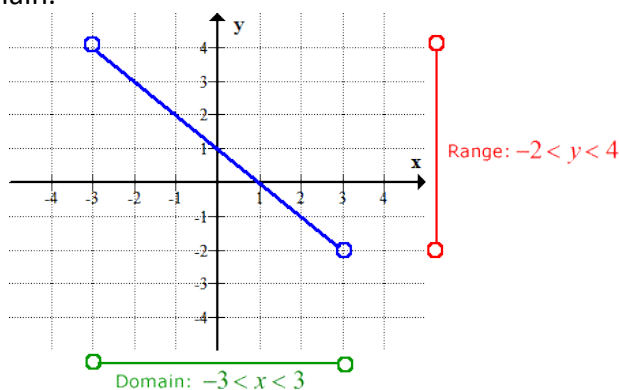
32. Find the solution to the following system of linear equations:
- $$3x + 6y = 9$$
- $$x = -2y + 3$$

MORE ON FUNCTIONS, DOMAIN, AND RANGE

Remember that a function has a domain and a range. The domain is the set of all possible x-values which will make the function "work", and will output real y-values. The range is the set of all possible y-values which will make the function "work." Some functions have a beginning point and an end point. In these cases, the domain is simply the x values where it begins and where it ends.

Some functions don't have endpoints. For example, a line that goes on forever in both directions. In this case, the domain (or x values) is the set of all real numbers because any x will work. The range (or y values) is also the set of all real numbers.

There are other cases where a function has certain places where there are holes in the domain and/or range. This is usually when a function is a fraction and the denominator is zero. We cannot divide by 0, therefore the denominator can never be 0. At those points where the denominator would be 0, the function is undefined. These points are not in the domain.



In the function $f(x) = \frac{x+4}{(x-1)(x+8)}$, for example, there are going to be some holes.

$\frac{x+4}{(x-1)(x+8)}$ If $x = 1$, for example, we would have a problem.

$\frac{1+4}{(1-1)(1+8)}$ Plug in 1 for all the x values

$\frac{5}{(0)(9)}$ Simplify

$\frac{5}{0}$ The denominator can NEVER be 0 so this point is undefined. In this equation, $x \neq 1$.

Similarly $x \neq -8$.

$\frac{x+4}{(x-1)(x+8)}$ Begin with the original equation

$\frac{-8+4}{(-8-1)(-8+8)}$ Plug in -8 for all x values

$\frac{-4}{(-9)(0)}$ Simplify

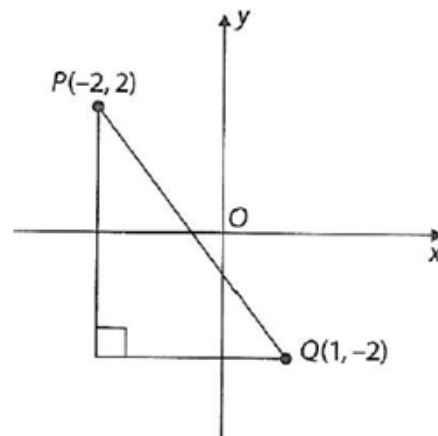
$\frac{-4}{0}$ The denominator can NEVER be 0 so this point is undefined. In this equation, $x \neq -8$.

The domain for this function is all real numbers except 1 and -8.

FINDING THE DISTANCE BETWEEN TWO POINTS

To find the distance between two points, use the Pythagorean theorem. The difference between x_1 and x_2 is one leg and the difference between y_1 and y_2 is the other leg.

For example, in the figure below we have two points $(-2,2)$ and $(1,-2)$. To find the distance between them, we make a right triangle. The difference between x_1 (which is -2) and x_2 (which is 1) equals 3 , so 3 is the length of the base leg. The difference between y_1 (which is 2) and y_2 (which is -2) equals 4 , so the length of the other leg is 4 . Now we have two sides of a right triangle and we can use the Pythagorean theorem to find the length of the hypotenuse.



$$a^2 + b^2 = c^2$$

$$3^2 + 4^2 = c^2$$

$$25 = c^2$$

$$5 = c$$

So the distance between point $(-2,2)$ and $(1,-2)$ is 5 .

If you prefer, rather than work through the Pythagorean theorem, you may memorize the

$$\text{distance formula } d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

which is just the points plugged into the Pythagorean theorem.

Sample Questions:

33. What is the distance between the points $(3,6)$ and $(5,-2)$?

A. 68

B. 10

C. $8\sqrt{2}$

D. $2\sqrt{17}$

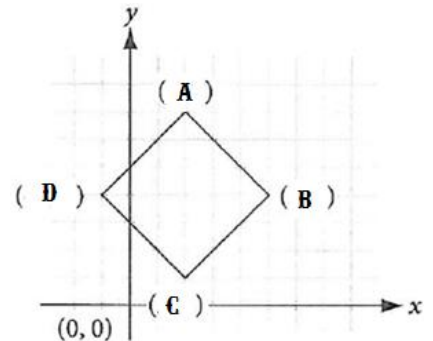
E. $2\sqrt{3}$

34. How many units apart are the points $P(-1, -2)$ and $Q(2,2)$ in the standard (x,y) coordinate plane?

35. Point $M(2,3)$ and point $N(6,5)$ are points on the coordinate plane. What is the length of the segment MN ?

36. Points A and B lie in the standard (x,y) coordinate plane. The (x,y) coordinates of A are $(2,1)$, and the (x,y) coordinates of B are $(-2,-2)$. What is the distance from A to B ?

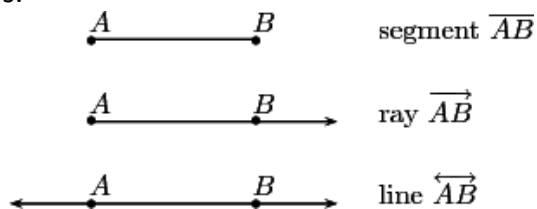
37. What is the distance, in coordinate units, between points J(-5,4) and K (6,-2) in the standard (x,y) coordinate plane?
38. Point A (-2,9) and point B (3, -3) are points on the coordinate plane. What is the length of the segment AB?
39. On a sonar map in the standard (x,y) coordinate plane, the Yellow Submarine and the Sandwich Submarine are located at the points (-7,4) and (-2,6), respectively. Each unit on the map represents an actual distance of 5 nautical miles. Which of the following is closest to the distance, in nautical miles, between the two submarines?
- A. 5
B. 19
C. 27
D. 30
E. 67
40. What is the area, in square units, of the square ABCD whose vertices are located at (2,7), (5,4), (2,1) and (-1, 4) respectively?



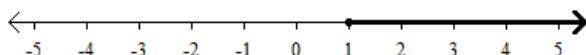
41. Points W (-2, 2), X (2, 2), and Y (2, -2) lie in the standard (x,y) coordinate plane and are 3 of the vertices of square WXYZ. What is the length, in coordinate units, of diagonal XZ?
- A. 2
B. 4
C. 16
D. $2\sqrt{2}$
E. $4\sqrt{2}$
42. Points A (-1, -1), B (-1, 2), and C (2, 2) lie in the standard (x,y) coordinate plane and are 3 of the vertices of square ABCD. What is the length, in coordinate units, of diagonal BD?
43. Points P (-1, 0), Q (1, 0), and R (1, -2) lie in the standard (x,y) coordinate plane and are 3 of the vertices of square PQRS. What is the length, in coordinate units, of diagonal PR?

MORE ON THE NUMBER LINE: LINES, SEGMENTS, AND RAYS

A line continues forever in both directions. To show this, lines have arrows at both ends. A line segment, often simply called a "segment," begins at one point and ends at another point. A ray begins at one point, but continues indefinitely in one direction. To show this a ray has an arrow at one end. With rays the direction matters.

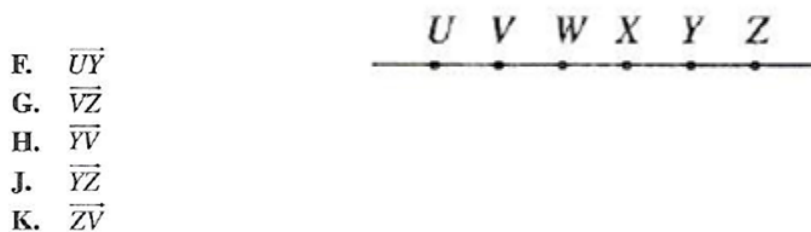


The ray below includes the points 1, 2, 3, ... and continues on forever. It does not contain -5, 0, $\frac{1}{2}$, 0.9 or any other number less than 1.

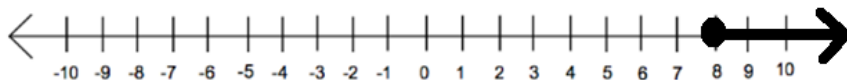


Sample Questions:

44. Six points (U, V, W, X, Y, Z) appear on a number line in that order, as shown in the figure below. Which of the following rays does NOT contain the line segment WX?

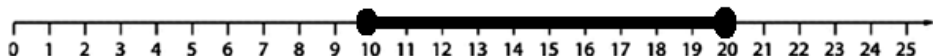


Rays are often used to show inequalities. A single variable may have more than one possible answer. For example, if I want a job that pays at least \$8.00 an hour, I can write that as $x \geq 8$ where x represents an acceptable wage. To show that on a number line, it would look like this:

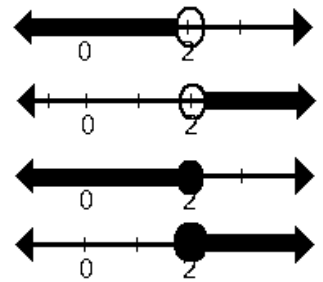


This means that any number greater or equal to 8 would be acceptable. I don't mind if I get paid more than \$8 per hour, in fact I would prefer it. As another example, I need to buy a Mother's day gift for my mom. I only have a \$20 bill. I don't want to spend less than \$10 because Dad said that if I didn't get her a nice present I will be grounded for a week. So the acceptable range for the price of the gift is between \$10 and \$20. In math terms it would look like this:

$10 \leq x \leq 20$ and on a number line, it would look like this:

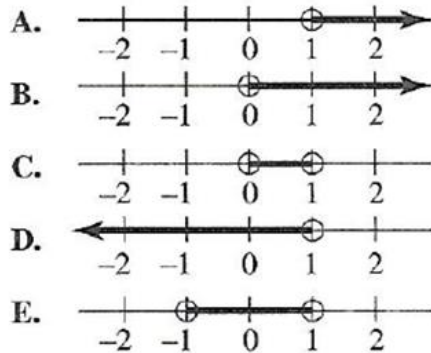


The endpoints are open circles for $<$ or $>$ and they are filled in for \leq or \geq . For example, in the figure below, the top inequality is $x < 2$. That means x could be anything less than 2, but 2 itself doesn't work. The next image shows $x > 2$. That means x could be anything greater than 2, but 2 itself doesn't work. The bottom two include 2 as an acceptable answer. $x \leq 2$ and $x \geq 2$.

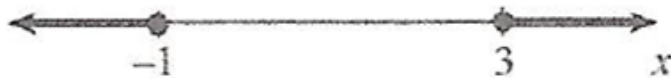


Sample Questions:

45. Which of the following graphs shows the inequality $x < 1$?

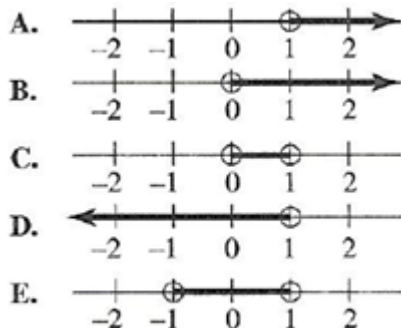


46. The number line graph below is the graph of which of the following inequalities?



- A. $-1 \leq x$ and $3 \leq x$
- B. $-1 \leq x$ and $3 \geq x$
- C. $-1 \leq x$ or $3 \leq x$
- D. $-1 \geq x$ or $3 \leq x$
- E. $-1 \geq x$ or $3 \geq x$

47. If y is an integer greater than 0 and x is an integer greater than 0 and $y > x$, which of the following graphs shows the set of all possible values for $\frac{y}{x}$?



MIDPOINT

In order to find the midpoint of 2 points on a number line, simply find the average. Add the two points and divide the sum by two.

$$\text{Midpoint} = \frac{\text{point 1} + \text{point 2}}{2}$$

Sample Questions:

48. What is the midpoint of -4 and 8 on the number line?

49. Point A is located at -7 on the real number line. If point B is located at -1, what is the midpoint of line segment AB?

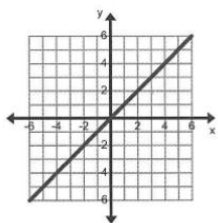
50. What is the midpoint of 3 and 7 on the number line?

51. Point R is located at -12 on the real number line. If point S is located at -1, what is the midpoint of line segment RS?

52. Point X is located at -15 on the real number line. If point Y is located at -11, what is the midpoint of line segment XY? (Hint: don't be confused by the names X and Y, we're still talking about points on a number line here and they can be called by any name)

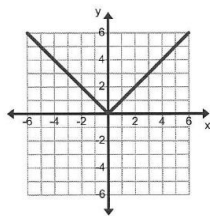
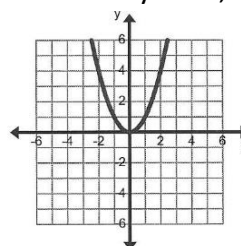
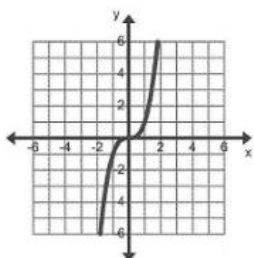
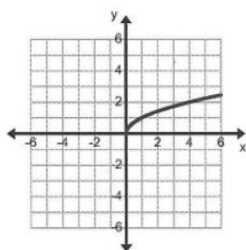
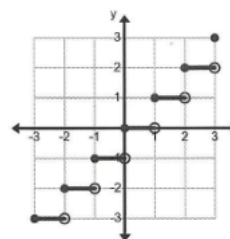
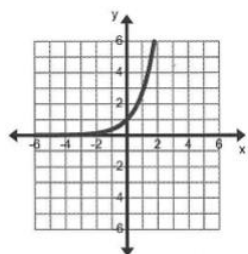
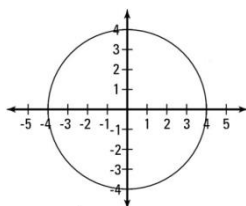
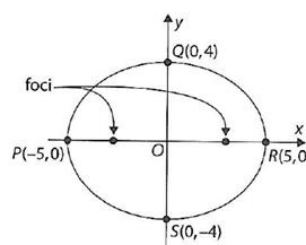
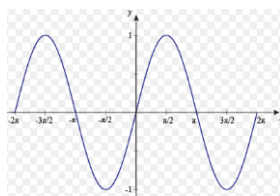
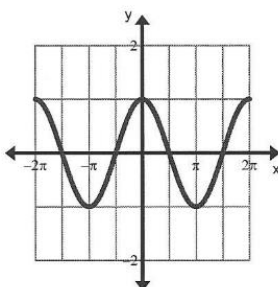
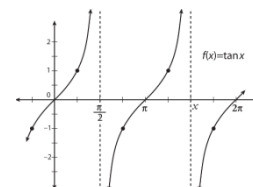
TYPES OF GRAPHS

So far we have talked about linear equations, which, as the name suggests, make lines. There are other types of equations which make different kinds of graphs. We'll introduce them briefly here, and use them later on.



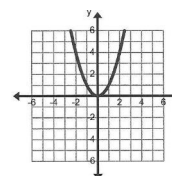
linear equation

$y = x$

absolute value equation $y = |x|$ quadratic equation $y = x^2$ cubic equation $y = x^3$ square root equation $y = \sqrt{x}$ step equation $y = \text{integer } x$ exponential equation $y = a^x$ circle equation $x^2 + y^2 = r^2$ ellipse equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ sine equation $y = \sin(x)$ cosine equation $y = \cos(x)$ tangent equation $y = \tan(x)$

Sample Questions:

53. What kind of equation makes a graph like the one shown at right?



54. True or false: Exponential functions will eventually outgrow all other functions.

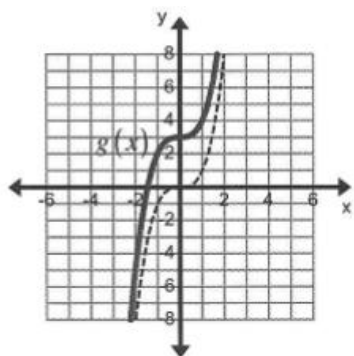
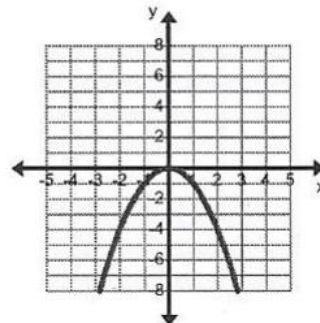
TRANSFORMATIONS OF EQUATIONS

More complex graphs are made from transforming one of those basic equations. Remember that a transformation is a general term for four specific ways to manipulate the shape of a point, a line, or a shape. The types of transformations in math are called: translation reflection, rotation, and dilation.

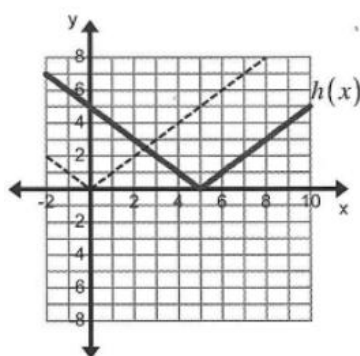
In other words a graph is either slid, or flipped, or turned, or stretched, or shrunk or a combination of some of these to make a new graph.

Example: This graph is made from a transformation of what kind of equation?

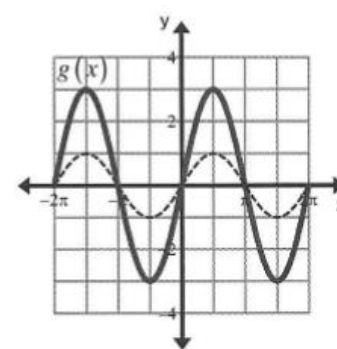
Answer: quadratic (it's just flipped, or reflected, around the x-axis)



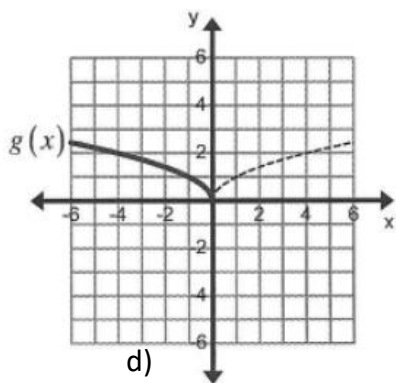
a)



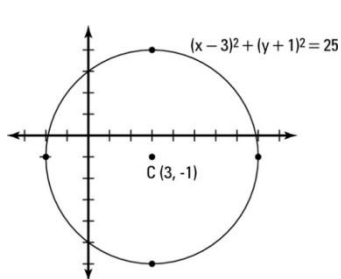
b)



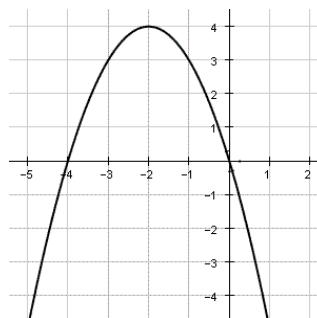
c)



d)



e)



f)

Sample Questions:

55. Which of the graphs above shows a transformation of a cubic equation?

56. Which of the graphs above shows a transformation of a sine equation?

57. Which of the graphs above shows a transformation of an absolute value equation?

ANSWERS

1. 1
2. $\frac{8}{7}$
3. $\frac{-1}{3}$
4. 3
5. E
6. 3
7. 8
8. $\frac{8}{5}$
9. $\frac{5}{3}$
10. $y = -2x - 1$
11. $y = \frac{3}{4}x - \frac{3}{4}$
12. $y = -x + 3$
13. $y = x + 1$
14. $y = \frac{8}{7}x + \frac{2}{7}$
15. True
16. False
17. False
18. False
19. B
20. D
21. $y = \frac{2}{3}x - 1$
22. -8
23. 2
24. B
25. (4, 2)
26. (3, -2)
27. (1, 2)
28. $(\frac{7}{4}, \frac{21}{8})$
29. $(\frac{-7}{2}, \frac{3}{2})$
30. $(1, \frac{1}{2})$
31. E
32. Infinite solutions
33. D
34. 5
35. $2\sqrt{5}$
36. 5
37. $\sqrt{157}$ or 12.53
38. 13
39. C
40. 18
41. E
42. $3\sqrt{2}$
43. $2\sqrt{2}$
44. J
45. D
46. D
47. A
48. 2
49. -4
50. 5
51. $\frac{-13}{2}$ or -6.5
52. -13
53. quadratic
54. true
55. a
56. c
57. b