

Name: \_\_\_\_\_

Date: \_\_\_\_\_

## Math 1 Basic Operations

### NUMBER DEFINITIONS

#### UNDEFINED

On the ACT, when something is **divided by zero**, it is considered *undefined*. For example, the expression  $\frac{a}{bc}$  is undefined if either  $b$  or  $c$  equals 0.

#### REAL/IMAGINARY

A real number is a number that has a **location on the number line** - positive, negative, whole numbers, fractions - all are considered real numbers. Imaginary numbers, on the other hand, are numbers that involve the square root of a negative number. The only numbers that aren't real are imaginary—a negative number under an even root. Also, complex numbers like  $i = \sqrt{-1}$  are imaginary.

#### INTEGER/NONINTEGER

Integers are like **whole numbers**, which means no fractions, decimals, or mixed numbers; however, they also include both negative and positive numbers, as well as zero.

**Example:** Which of the following expressions must be an even integer if  $x$  is an integer?

- a.  $x + 5$
- b.  $\frac{x}{4}$
- c.  $x^4$
- d.  $4x$
- e.  $5^x$

**Solution:** Because of the words “must be”, the expression given must produce an even number in every instance. If you can find one instance in which an expression produces an odd number, you can eliminate that expression and answer choice. Since  $x$  can be any integer, there are many combinations you can try. In (A),  $x$  could be 4, in which case,  $x + 5 = 4 + 5 = 9$ , which is not even. In (B),  $x$  could be 1, which would make  $x/4 = \frac{1}{4}$ , which is not even (nor is it an integer). If  $x = 1$  (C) gives  $x^4 = 1$ , which is not even; in (E), 5 raised to any integer power will always be odd. Only (D) generates an even number with every integer. This is because the product of an even number multiplied by either an even or an odd number will always be even.

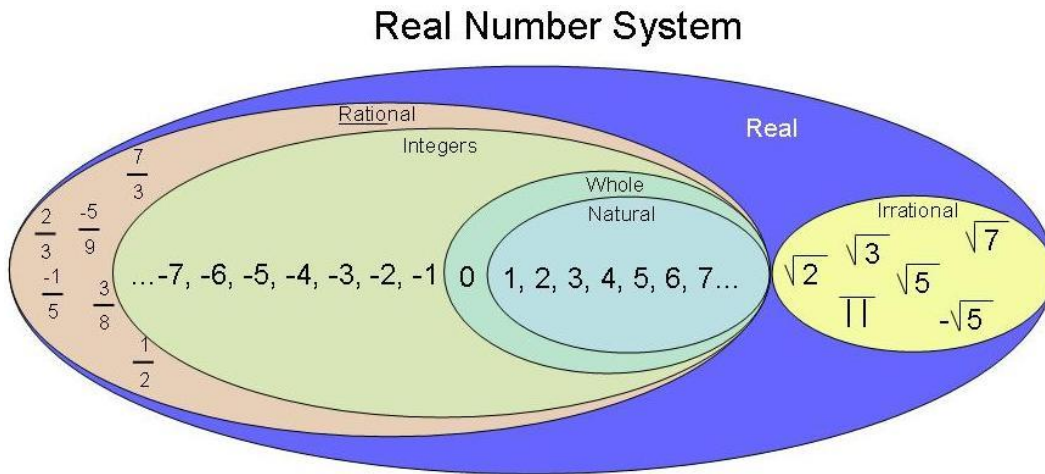
#### RATIONAL/IRRATIONAL

Any number that can be expressed as a ratio, fraction or a repeating decimal is a rational number. Rational contains the word ratio. This includes numbers like 3,  $\frac{2}{5}$ ,  $-.1666$ , or  $.333$ . **Irrational numbers cannot be expressed precisely as a fraction or decimal.** For the purposes of the ACT, the most important **irrational numbers** are  $\sqrt{2}$ ,  $\sqrt{3}$ , and  $\pi$ .

**Example:** All of the following statements about rational and/or irrational numbers must be true EXCEPT:

- a. The sum of any two rational numbers is rational
- b. The product of any two rational numbers is rational
- c. The sum of any two irrational numbers is rational
- d. The product of a rational and an irrational number may be rational or irrational
- e. The product of any two irrational numbers is irrational

**Solution:** On a “must be” question, the relationship must be true in all cases. If there is one case in which a relationship is not true, that answer choice is incorrect. Because this is an EXCEPT question, the incorrect relationship will be the correct answer. Choice (E) is often true, but not always. If you multiply  $\sqrt{2}$ , an irrational number, by itself, you get a rational number: 2.



### **CALCULATORS IN WORKING WITH RATIONAL AND IRRATIONAL NUMBERS**

To determine if a number is rational or irrational, it can help to try to convert into a fraction. In the MATH menu, the first item is >Frac. This will convert decimal to a fraction if the number is rational.

Sample Questions:

Identify each as rational or irrational.

1. 0.375
2. 0.166666666666 (until the end of the screen)
3.  $\pi$
4. 0.479109801431

### **POSITIVE/NEGATIVE**

All numbers greater than 0 are positive and all numbers less than 0 are negative. Zero is neither positive nor negative.

### **ODD/EVEN**

An odd number is an integer which is not a multiple of two. If it is divided by two the result is a fraction. One is the first odd positive number. An even number is an integer which is "evenly divisible" by two. This means that if the integer is divided by 2, it yields no remainder. Zero is an even number because zero divided by two equals zero. Odd and Even numbers can be either positive or negative.

## Sample Questions:

Use these numbers to answer the following questions.

-81, -19, -9, -6, -1/4, -0.15, 0, 1.75, 2, 3, 12, 16, 81

5. List all the positive numbers.
6. List all the negative integers.
7. List all the odd integers.
8. List all the even integers.
9. List all the positive, even integers in consecutive order.
10. List all the numbers that are neither positive nor negative.
11. Which of the following is a rational number?

- a.  $\sqrt{2}$
- b.  $\sqrt{\pi}$
- c.  $\sqrt{7}$
- d.  $\sqrt{\frac{5}{25}}$
- e.  $\sqrt{\frac{64}{49}}$

**SCIENTIFIC NOTATION**

Scientific notation is the way that scientists easily handle very large numbers or very small numbers. For example, instead of writing 0.0000000056, write  $5.6 \times 10^{-9}$ . Think of  $5.6 \times 10^{-9}$  as the product of two numbers: 5.6 (the digit term) and  $10^{-9}$  (the exponential term).

Examples of scientific notation.

$10000 = 1 \times 10^4$	$24327 = 2.4327 \times 10^4$
$1000 = 1 \times 10^3$	$7354 = 7.354 \times 10^3$
$100 = 1 \times 10^2$	$482 = 4.82 \times 10^2$
$10 = 1 \times 10^1$	$89 = 8.9 \times 10^1$ (not usually done)
$1 = 10^0$	
$1/10 = 0.1 = 1 \times 10^{-1}$	$0.32 = 3.2 \times 10^{-1}$ (not usually done)
$1/100 = 0.01 = 1 \times 10^{-2}$	$0.053 = 5.3 \times 10^{-2}$
$1/1000 = 0.001 = 1 \times 10^{-3}$	$0.0078 = 7.8 \times 10^{-3}$
$1/10000 = 0.0001 = 1 \times 10^{-4}$	$0.00044 = 4.4 \times 10^{-4}$

The exponent of 10 is the number of places the decimal point must be shifted to give the number in long form. A **positive** exponent shows that the decimal point is shifted that number of places to the right. A **negative** exponent shows that the decimal point is shifted that number of places to the left. In scientific notation, the digit term indicates the number of significant figures in the number. The exponential term only places the decimal point.

**Example:**  $46600000 = 4.66 \times 10^7$

**Solution:** This number only has 3 significant figures. The zeros are not significant; they are only holding a place.

**Example:**  $0.00053 = 5.3 \times 10^{-4}$

**Solution:** This number has 2 significant figures. The zeros are only place holders.

Sample Questions:

12. If there are  $8 \times 10^{12}$  hydrogen molecules in a volume of  $4 \times 10^4$  cubic centimeters, what is the average number of hydrogen molecules per cubic centimeter?

13. Which of the following numbers is the least in value?

- a.  $0.02 \times 10^4$
- b.  $0.2 \times 10^3$
- c.  $2 \times 10^{-2}$
- d.  $20.0 \times 10^2$
- e.  $0.002 \times 10^5$

14. The number 0.002 is 100 times as large as which of the following numbers?

- a. 0.000002
- b. 0.00002
- c. 0.0002
- d. 0.02
- e. 0.2

15. Which of the following is equivalent to  $5.046 \times 10^{-3}$

- a. 5,046
- b. 504.6
- c. 5.046
- d. 0.05046
- e. 0.005046

## REDUCING FRACTIONS

To reduce a fraction to lowest terms, **factor out and cancel** all factors the numerator and denominator have in common. For instance, if you had the fraction  $28/36$ , you could reduce this fraction, because both 28 and 36 have a factor of 4 that can be taken out:

$$\frac{28}{36} = \frac{4 \times 7}{4 \times 9} = \frac{7}{9}$$

## ADDING/SUBTRACTING FRACTIONS

To add or subtract fractions, first find a **common denominator**, and then add or subtract the numerators. Finding a common denominator often involves multiplying one or more of the fractions by a number so that the denominators come out the same:

$$\frac{2}{15} + \frac{3}{10} = \frac{4}{30} + \frac{9}{30} = \frac{4+9}{30} = \frac{13}{30}$$

## MULTIPLYING FRACTIONS

To multiply fractions, **multiply** straight across—numerator times numerator and denominator times denominator. Remember to simplify or reduce equation to lowest terms.

$$\frac{5}{7} \times \frac{3}{4} = \frac{5 \times 3}{7 \times 4} = \frac{15}{28}$$

## DIVIDING FRACTIONS

To divide fractions, **invert** the fraction in the denominator and **multiply**.

$$\frac{1}{2} \div \frac{3}{5} = \frac{1}{2} \times \frac{5}{3} = \frac{1 \times 5}{2 \times 3} = \frac{5}{6}$$

## FRACTION PROBLEMS

**Example:** For how many integer values of  $x$  will  $\frac{7}{x}$  be greater than  $\frac{1}{4}$  and less than  $\frac{1}{3}$ ?

- a. 6
- b. 7
- c. 12
- d. 28
- e. Infinitely many

**Solution:** For  $7/x$  to be greater than  $1/4$ , the denominator  $x$  has to be less than 4 times the numerator, or 28. And for  $7/x$  to be less than  $1/3$ , the denominator  $x$  has to be greater than 3 times the numerator, or 21. Thus,  $x$  could be any of the integers 22 through 27, of which there are 6.

## CONVERTING A MIXED NUMBER TO AN IMPROPER FRACTION

To convert a mixed number, which is a whole number with a fraction, to an improper fraction, which is a fraction where the numerator is bigger than the denominator, **multiply** the whole number part by the denominator, then **add** the numerator. The result is the new numerator (over the same denominator). To convert  $7\frac{1}{2}$  first multiply 7 by 3, then add 1 to get the new numerator of 22. Put that over the same denominator, 3, to get  $\frac{22}{3}$ .

Sample Questions:

16.  $3\frac{4}{9} =$

17.  $6\frac{5}{6} =$

18.  $9\frac{7}{12} =$

19.  $4\frac{2}{3} =$

20.  $4\frac{8}{9} =$

21.  $8\frac{3}{7} =$

### CONVERTING AN IMPROPER FRACTION TO A MIXED NUMBER

To convert an improper fraction to a mixed number, **divide** the denominator into the numerator, and the remainder will be the numerator of the fraction part, with the same denominator.

Example, convert  $\frac{108}{5}$ , first divide 5 into 108, which yields 21 with a remainder of 3. Therefore,  $\frac{108}{5} = 21\frac{3}{5}$ .

Sample Questions:

22.  $38/8 =$

23.  $30/4 =$

24.  $31/6 =$

25.  $31/8 =$

26.  $9/2 =$

27.  $19/5 =$

Sample Questions for working with fractions:

28.  $\frac{1}{4} \times \frac{2}{5} \times \frac{3}{6} \times \frac{4}{7} \times \frac{5}{8} \times \frac{6}{9} \times \frac{7}{10} = ?$

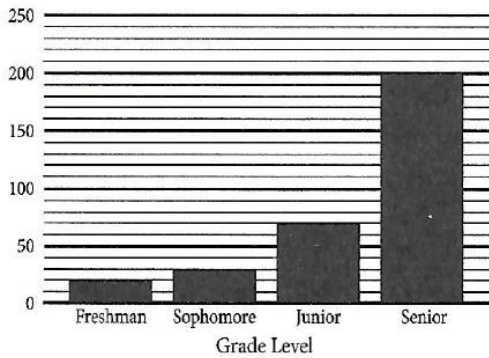
29.  $5\frac{5}{6} + 3\frac{5}{8} + 1\frac{1}{3} ?$

30. The following is equal to  $\frac{3-\frac{1}{2}}{2+\frac{3}{4}} ?$

31.  $\frac{8+\frac{1}{3}}{2+\frac{1}{9}}$  is equivalent to:

32. Tanya used  $3\frac{3}{8}$  yards of fabric to make her dress, and she used  $1\frac{1}{3}$  yards of fabric to make her jacket. What is the total amount, in yards, that Tanya used for the complete outfit of dress and jacket?

33. The following bar graph shows the number of people at the spring prom, according to their grade level at the high school. According to the graph, what fraction of the people at the prom were sophomores?



Number of Prom Attendees, by Grade Level

### COMPARING FRACTIONS

One way to compare fractions is to manipulate them so they have a **common denominator**. For instance, to compare  $\frac{3}{4}$  and  $\frac{5}{7}$ :

$$\frac{3}{4} = \frac{21}{28} \text{ and } \frac{5}{7} = \frac{20}{28}; \frac{21}{28} \text{ is greater than } \frac{20}{28}, \text{ so } \frac{3}{4} \text{ is greater than } \frac{5}{7}.$$

Another way to compare fractions is to convert them both to **decimals**.  $\frac{3}{4}$  converts to approximately .75.

Sample Questions:

34. What rational number is halfway between  $\frac{1}{6}$  and  $\frac{1}{2}$ ?

35. What rational number is halfway between  $\frac{1}{5}$  and  $\frac{1}{3}$ ?

36. What fraction lies exactly halfway between  $\frac{1}{3}$  and  $\frac{1}{2}$ ?

37. What fraction lies exactly halfway between  $\frac{2}{3}$  and  $\frac{3}{4}$ ?

## CONVERTING FRACTIONS TO DECIMALS

To convert a fraction to a decimal, **divide the numerator by the denominator**. To convert  $5/8$ , divide 5 by 8, yielding .625. Often, these numbers will start repeating, such as  $1/6$ . When 1 is divided by 6, the decimal starts repeating almost right away, so it can be written as 1.66 (the line over the 66 means "repeating").

**Example:** As a decimal, what is the sum of  $\frac{2}{3}$  and  $\frac{2}{12}$ ?

**Solution:** Normally you would have a choice--either convert to fractions to decimals first and then add, or add the fractions first and then convert the sum to a decimal. In this case, however, both fractions would convert to endlessly repeating decimals, which would be unwieldy when adding. In this case, it makes sense to add first, then convert.  $2/3$  is  $8/12$ . So adding  $8/12$  to  $2/12$  becomes  $10/12$  and reducing becomes  $5/6$  or 0.833.

Sample Questions:

38. What is the sum of  $\frac{3}{16}$  and 0.175?

39.  $\frac{2}{3} + \frac{1}{12} =$  what decimal number?

## REMAINDERS

The remainder is the number left over after division. If you divided 48 by 5, you'd first find the nearest multiple of 5 to 48, which is 45 ( $9 \times 5$ ). This means 5 goes into 48 nine times, but then there is still 3 left over, which makes the remainder 3.

Sample Questions:

40. How many times does 2 go into 15 evenly?

41. How much is left over?

42. What is 15 divided by 2?



## WORKING WITH EXPONENTS AND ROOTS

Exponents are shorthand for repeated multiplication of the same thing by itself. For instance, the shorthand for multiplying three copies of the number 5 is shown on the right-hand side of the "equals" sign in  $(5)(5)(5) = 5^3$ . The "exponent", being 3 in this example, stands for however many times the value is being multiplied. The thing that's being multiplied, being 5 in this example, is called the "base".

This process of using exponents is called "raising to a power", where the exponent is the "power". The expression " $5^3$ " is pronounced as "five, raised to the third power" or "five to the third". There are two specially-named powers: "to the second power" is generally pronounced as "squared", and "to the third power" is generally pronounced as "cubed". So " $5^3$ " is commonly pronounced as "five cubed".

The "opposite" of exponents are roots. The square root of a number is a value that, when multiplied by itself, gives the number. Example:  $4 \times 4 = 16$ , so a square root of 16 is 4. Note that  $(-4) \times (-4) = 16$  too, so  $-4$  is also a square root of 16. The symbol is  $\sqrt{\quad}$  which always means the positive square root. Example:  $\sqrt{36} = 6$  (because  $6 \times 6 = 36$ )

Sample Questions:

43.  $3^4 = ?$

44.  $\sqrt{25} = ?$

45.  $7^2 = ?$

46.  $\sqrt{81} = ?$

## ADDING/SUBTRACTING SIGNED NUMBERS

To **add a positive and a negative**, subtract the negative number from the positive number. For instance,  $-9 + 1$  is the same as  $1 - 9$ . Alternatively, you can find the difference between the two numbers, and then keep the original sign of the larger number. Again using the example  $-9 + 1$ , the difference between 1 and 9 is 8, and since 9 was the larger number, keep the negative sign, so  $-9 + 1 = -8$ .

**Subtracting** a negative number is the same as adding a positive. For example, think of  $17 - (-21)$  as  $17 + (+21)$ .

To **add or subtract a string of positives and negatives**, add up all of the positive numbers, and add up all of the negative numbers. Then add those two sums together, keeping the sign of the greater sum. For instance, if you had the string  $8 + 3 - 12 + 4 - 6 - 5$ , combine the positives:  $8 + 3 + 4 = 15$ . Combine the negatives:  $-12 + (-6) + (-5) = -23$ . Now combine the two sums:  $15 + (-23) = -8$ .

Sample Questions:

47.  $24 - (-10) + (-30) = ?$

48.  $8 + (-3) - (+5) + (-8) = ?$

**MULTIPLYING/DIVIDING SIGNED NUMBERS**

Positives multiplied/divided with other positives are always positive. An even number of negative numbers, when multiplied or divided, will also be positive. However, if there is an odd number of negatives, the multiplication/division will be negative.

For instance,  $(-2) \times (-3) = 6$ , but  $(-2) \times (-3) \times (-4) = -24$ .

Sample Questions:

49. What is  $\sqrt{1}$ ?

50. What is  $-1 \times -1$ ?

51. What is  $-1 \times -1 \times -1$ ?

52. What is  $\sqrt{-1}$ ?

53. What real number could you square to get a product of -1?

54. What is  $9 \times (-4) + 6 \times (-3)$ ?

55. What is  $15/-3$ ?

**ORDER OF OPERATIONS (PEMDAS)**

When performing multiple operations, remember **PEMDAS**, which means start with anything inside **Parentheses** first, then deal with any **Exponents**, then all of the **Multiplication** and **Division** (left to right), and finally all of the **Addition** and **Subtraction** (left to right).

In the expression  $9 - 2 \times (5 - 3)^2 + 6 \div 3$ , begin with the parentheses:  $(5 - 3) = 2$ . Then do the exponent:  $2^2 = 4$ , so now the expression is:  $9 - 2 \times 4 + 6 \div 3$ . Next do the multiplication,  $2 \times 4 = 8$ , and division,  $6 \div 3 = 2$ , to get  $9 - 8 + 2$ . Finally, subtraction,  $9 - 8 = 1$ , and then the addition,  $1 + 2$ , which equals 3.

Sample Questions:

$$56. \{[2^3 + 4] \cdot 2\} - \frac{3}{3} + \frac{10}{5}$$

$$57. \frac{\frac{\frac{3}{2} + \frac{7}{4}}{(\frac{15}{8} - \frac{3}{4}) - (\frac{4+3}{-4+3})}}$$

$$58. \frac{3 + (-3)(5) + (8)(4)}{-5(8) + 3(-5)(-4) - (3) + (-7 \times 5)}$$

$$59. \frac{\frac{\frac{1}{2} + \frac{3}{4}}{(\frac{7}{6} - \frac{3}{4}) \times (\frac{2+8}{-1+9})}}$$

$$60. \frac{\frac{\frac{2+8}{-1+9}}{(\frac{7}{6} - \frac{3}{4}) \times (\frac{1}{2} + \frac{3}{4})}}$$

### MULTIPLES OF 2 AND 4

An integer is divisible by 2 if the **last digit is even**. An integer is divisible by 4 if the **last two digits form a multiple of 4**. The last digit of 562 is 2, which is even, so 562 is a multiple of 2. The last two digits make 62, which is not divisible by 4, so 562 is not a multiple of 4. However, 4,928 is divisible by 4 because 28 is divisible by 4.

### MULTIPLES OF 3 AND 9

An integer is divisible by 3 if the **sum of its digits is divisible by 3**. An integer is divisible by 9 if the **sum of its digits is divisible by 9**. Take the number 957-the sum of the digits is  $9 + 5 + 7 = 21$ , and 21 is divisible by 3 but 21 is not divisible by 9, so 957 is divisible by 3 but not 9.

### MULTIPLES OF 5 AND 10

An integer is divisible by 5 if the **last digit is 5 or 0**. An integer is divisible by 10 if the **last digit is 0**. The last digit of 665 is 5, so 665 is a multiple of 5 but not a multiple of 10.

Sample Problems:

61. If an integer is divisible by 6 and by 9, then the integer must be evenly divisible by which of the following?

- I. 12
- II. 18
- III. 36
- a. I only
- b. II only
- c. I and II only
- d. I, II and III
- e. None

### FACTOR/MULTIPLE

A factor of an integer is any number that divides precisely into that integer (with no remainder). The multiple of an integer is that integer times any number. In other words, factor  $\times$  factor = multiple. All numbers are both factors and multiples of themselves. Six is a factor of 12, and 24 is a multiple of 12. Twelve is both a factor ( $12 \div 12 = 1$ ) and a multiple ( $12 \times 1 = 12$ ) of itself.

**Example:** Which of the following is NOT a factor of 1,776?

- a. 12
- b. 16
- c. 18
- d. 24
- e. 37

**Solution:** When dealing with NOT questions, eliminate answer choices that work. Use your calculator and try out the answer choices.  $1,776 \div 12 = 148$ , and because 12 is a factor of 1,776, you can eliminate (A). The same is true of the numbers in (B), (D), and (E). However,  $1,776 \div 18 = 98.\overline{66}$ , so 18 is NOT a factor of 1,776 and is the correct answer.

Sample Questions:

62. What is the greatest integer that is a factor of both  $8 \times 10^9$  and  $6 \times 10^3$ ?

## PRIME FACTORIZATION

A **prime number** is a positive integer that is divisible without a remainder by only 1 and itself. The number 2 is the smallest prime number and the only even prime number; 1 is not considered prime. To find the prime factorization of an integer, use a factor tree to keep breaking the integer up *into* factors until **all the factors are prime**.

**Example:** What are the prime factors of 36?

**Solution:**  $36 = 4 \times 9 = 2 \times 2 \times 3 \times 3$

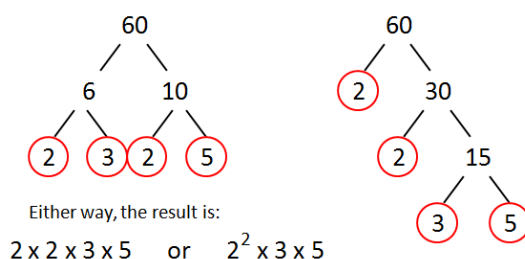
Sample Question:

63. How many distinct prime factors does the number 36 have?

## FACTOR TREE

An easy way to find factors is through a factor tree. Write the number at the top and diagonal lines under with factors of the original number. Continue until arrive at prime numbers on all “branches”.

**Example:** Factors of the number 60.



Sample Question:

64. Make a factor tree for the number 48.

## RELATIVE PRIMES

Two integers are relative primes if they share no prime factors. To determine whether two integers are relative primes, break them both down to their prime factorizations. For example:  $35 = 5 \times 7$  and  $54 = 2 \times 3 \times 3$ . They have **no prime factors in common**, so 35 and 54 are relative primes.

## COMMON MULTIPLE

You can always get a common multiple of two numbers by **multiplying** them, but unless the two numbers are relative primes, the product will not be the least common multiple the smallest multiple both of those numbers divide into. For example, to find a common multiple for 12 and 15, you could just multiply:  $12 \times 15 = 180$ .

**Example:** How many positive integers less than 50 are multiples of 4 but NOT multiples of 6?

- a. 4
- b. 6
- c. 8
- d. 10
- e. 12

**Solution:** The safest strategy is simply to list out the possibilities. It's also helpful to realize that multiples of both 4 and 6 are multiples of 12 (the least common multiple between the two), so skip over all multiples of 12:

4, 8, ~~12~~, 16, 20, ~~24~~, 28, 32, ~~36~~, 40, 44, 48

So there are 8 in all (C).

### LEAST COMMON MULTIPLE (LCM)

To find the least common multiple, check out the **multiples of the larger number** until you find one that's **also a multiple of the smaller**.

To find the LCM of 12 and 15, begin by taking the multiples of 15:  $1 \times 15 = 15$ , which is not divisible by 12;  $2 \times 15 = 30$ , not divisible by 12; nor is 45, which is  $3 \times 15$ . But the next multiple of 15,  $4 \times 15 = 60$ , is divisible by 12, so it's the LCM.

**Example:** What is the least common multiple of 25, 16, and 40?

- a. 27
- b. 32
- c. 320
- d. 400
- e. 16,000

**Solution:** Use answer choices and work backwards. Find the smallest number that divides evenly by 16, 25, and 40. This is 400:  $400 \div 16 = 25$ ;  $400 \div 25 = 16$ ; and  $400 \div 40 = 10$ . If you selected (E), be careful – this is a common multiple of all three numbers, but it is not the *least* common multiple.

Sample Questions:

65. What is the least common multiple of 70, 60, and 50?

66. What is the least common multiple of 3,  $4a$ ,  $5b$  and  $6ab$ ?

67. When the choir is arranged in rows of 5 people each, the last row is one person short. When the choir is arranged in rows of 6 people each, the last row is still one person short. What is the least possible number of people in the choir?

## GREATEST COMMON FACTOR (GCF)

The greatest common factor of two numbers is the highest number that divides precisely into each of them without a remainder. To find the greatest common factor, break down both numbers into their prime factorizations, and take **all the prime factors they have in common**. For example, take 36 and 48:  $36 = 2 \times 2 \times 3 \times 3$  and  $48 = 2 \times 2 \times 2 \times 2 \times 3$ . What they have in common is two 2s and one 3, so the GCF is  $2 \times 2 \times 3 = 12$ .

**Example:** Given that  $a$  and  $b$  are positive integers, and the greatest common factor of  $a^4b^2$  and  $a^3b$  is 54, what is a possible value for  $b$ ?

- a. 2
- b. 3
- c. 6
- d. 9
- e. 27

**Solution:** To find the greatest common factor, find all factor pieces that the two expressions have in common. In this case, the factors in common are  $a$ ,  $a$ ,  $a$ , and  $b$ , or  $a^3b$ . It is given that the greatest common factor is 54, so think of a cubic number that is a factor of 54. The first cubic numbers are  $1^3$  (1),  $2^3$  (8), and  $3^3$  (27). Twenty-seven is a factor of 54:  $27 \times 2 = 54$ , so a possible value of  $b$  is 2. Choice B is the value of the variable  $a$ . Choice C, 6, is a factor of 54, but that leaves the value of 9 for  $a$  and 9 is not a perfect cube. The same reasoning would eliminate D. Choice E is the value of  $a^3$ .

Sample Questions:

68. For all positive integers  $x$ , what is the greatest common factor of the 2 numbers  $216x$  and  $180x$ ?

69. If  $x$  and  $y$  are positive integers such that the greatest common factor of  $x^2y^2$  and  $xy^3$  is 27, then which of the following could  $y$  equal?

- a. 81
- b. 27
- c. 18
- d. 9
- e. 3

70. Two whole numbers have a greatest common factor of 8 and a least common multiple of 48. Which of the following pairs of whole numbers will satisfy the given conditions?

- a. 4 and 9
- b. 5 and 10
- c. 10 and 16
- d. 14 and 20
- e. 16 and 24

## ABSOLUTE VALUE

The absolute value of something is its distance from zero on the number line, which is why absolute value is always positive. Treat absolute value signs a lot like **parentheses**. Do what's inside them first and then take the absolute value of the result. Don't take the absolute value of each piece between the bars before calculating. In order to calculate  $|(-12) + 5 - (-4)| - |5 + (-10)|$ , first do what's inside the bars to get:  $|-3| - |-5|$ , which is  $3 - 5$ , or  $-2$ .

**INEQUALITY**

An inequality is like an equation that uses symbols for "less than"(<) and "greater than"(>) where an equation uses a symbol for "is equal to" (=). For example,  $5 > 3$  or  $14 < 48$ .

Sample Questions:

71.  $|7 - 3| - |3 - 7| = ?$

72.  $-3 |-6 + 8| = ?$

73. Is the following inequality true?

$-4 |-5 + 3| > -5 |-1 + 6|$

**RECIPROCAL**

The reciprocal of a fraction is the inverse of that fraction. To find the reciprocal of a fraction, switch the numerator and the denominator. The reciprocal of  $\frac{3}{7}$  is  $\frac{7}{3}$ . The reciprocal of 5 (or  $\frac{5}{1}$ , because all whole numbers are over 1) is  $\frac{1}{5}$ . The product of reciprocals is always 1.

Sample Questions:

74. What is the reciprocal of  $-\frac{1}{4}$ ?

75. What is the opposite reciprocal of 2?

76. When a number and its reciprocal are multiplied, what is the product?

77. What is the product of  $\frac{4}{5}$  and the reciprocal of  $\frac{8}{3}$ ?

**BASIC OPERATIONS VOCABULARY**

Sample Questions:

Use the following list of terms to answer the following questions:

Number, Integer, Positive, Negative, Even, Odd, Consecutive, Reciprocal, Remainder

78. Which terms describe the number 6?

79. Which terms describe the number  $-\frac{1}{5}$ ?

80. Which terms describe the number 0?



81. When 14 is divided by 3, it has a \_\_\_\_\_ of 2.
82. The \_\_\_\_\_ of  $-1/4$  is -4.
83. The numbers 2, 4, 6 are listed in \_\_\_\_\_ order, but the numbers 3, 1, 14 are not.

#### **RANDOM BASIC OPERATIONS QUESTIONS**

84. While working on a problem on his calculator, Tex had meant to multiply a number by 3, but he accidentally divided the number by 3. Which of the following calculations could Tex then do to the result on the screen in order to obtain the result he originally wanted?
- a. Multiply by 3
  - b. Multiply by 9
  - c. Divide by 3
  - d. Divide by 9
  - e. Add the original number
85. A carpenter is cutting wood to make a new bookcase with a board that is 12 feet long. If the carpenter cuts off three pieces, each of which is 17 inches long, how many inches long is the remaining fourth and final board? (A foot contains 12 inches.)
86. What is the value of  $117 - 54 + 6$ , rounded to the nearest ten?
87. Which of the following operations will produce the smallest result when substituted for the blank in the expression  $2/3$  \_\_\_\_\_ - 3?
- a. Plus
  - b. Minus
  - c. Multiplied by
  - d. Divided by
  - e. Averaged with

## Answer Key

1. Rational
2. Rational
3. Irrational
4. Rational
5. 1.75, 2, 3, 12, 16, 81
6. -81, -19, -9, -6
7. -81, -19, -9, 3, 81
8. -6, 0, 2, 12, 16
9. 2, 12, 16
10. 0
11.  $\sqrt{\frac{64}{49}}$
12.  $2 \times 10^8$
13.  $2 \times 10^{-2}$
14. 0.00002
15. 0.005046
16.  $31/9$
17.  $41/6$
18.  $115/12$
19.  $14/3$
20.  $44/9$
21.  $59/7$
22.  $4 \frac{3}{4}$
23.  $7 \frac{1}{2}$
24.  $5 \frac{1}{6}$
25.  $3 \frac{7}{8}$
26.  $4 \frac{1}{2}$
27.  $3 \frac{4}{5}$
28.  $1/120$
29. 10 19/24
30.  $10/11$
31.  $75/19$
32.  $4 \frac{17}{24}$
33.  $3/32$
34.  $1/3$
35.  $4/15$

36.  $5/12$
37.  $17/24$
38. 0.3625
39. 0.75
40. 7
41. 1
42. 7 R1
43. 81
44. 5
45. 49
46. 9
47. 4
48. -8
49. 1
50. 1
51. -1
52. Imaginary
53. None
54. -54
55. -5
56.  $[(8 + 4) \cdot 2] - \frac{15}{15} + \frac{30}{15} = (12 \cdot 2) - \frac{45}{15}$   
 $= 24 - 1 + 2 = 25$
57.  $2/5$
58. 
$$\frac{3 + (-3)(5) + (8)(4)}{-5(8) + 3(-5)(-4) - (3) + (-7 \times 5)}$$

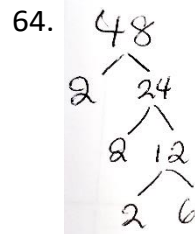
$$= \frac{3 - 15 + 32}{-40 + 60 - 3 - 35}$$

$$= \frac{20}{-18} = -\frac{10}{9} \text{ or } 1 \frac{1}{9}$$
59. 
$$\frac{\frac{5}{4}}{\left(\frac{5}{12}\right) \times \left(\frac{5}{4}\right)} = \frac{5}{4} \times \frac{48}{25} = \frac{12}{5} = 2 \frac{2}{5}$$
60.  $2 \frac{2}{5}$

61. II only

62.  $2 \times 10^3$ 

63. 2



65. 2100

66. 60ab

67. 29

68. 36x

69. 3

70. 16 and 24

71. 0

72. -6

73. True  $-8 > -25$ 

74. -4

75.  $-1/2$ 

76. 1

77.  $3/10$ 

78. Number, integer, positive, even

79. Number, negative

80. Number, integer, even

81. Remainder

82. Reciprocal

83. Consecutive

84. Multiply by 9

85. 93

86. 70

87. Plus