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## Math 1 Variable Manipulation Part 2 <br> Exponents \& Roots

## PRE-ALGEBRA REVIEW:

## WORKING WITH EXPONENTS

Exponents are shorthand for repeated multiplication of the same thing by itself. For instance, the shorthand for multiplying three copies of the number 5 is shown on the right-hand side of the "equals" sign in $(5)(5)(5)=5^{3}$. The "exponent", being 3 in this example, stands for however many times the value is being multiplied. The thing that's being multiplied, 5 in this example, is called the "base".

This process of using exponents is called "raising to a power", where the exponent is the "power". The expression " $5^{3 "}$ is pronounced as "five, raised to the third power" or "five to the third". There are two specially-named powers: "to the second power" is generally pronounced as "squared", and "to the third power" is generally pronounced as "cubed". So " $5^{3 "}$ is commonly pronounced as "five cubed".

When we deal with numbers, we usually just simplify; we'd rather deal with " 27 " than with " $3^{3 "}$. But with variables, we need the exponents, because we'd rather deal with "x" than with "xxxxxx".

Sample Questions:

1. What is the value of $4^{3}$ ?
2. What is the value of $8^{2}$ ?

## ADD AND SUBTRACT EXPONENTS

To add or subtract powers, both the variables and the exponents of the variables must be the same. Perform the required operations on the coefficients, leaving the variable and exponent as they are: $x^{2}+x^{3}+x^{3}=x^{2}+$ $2 x^{3}$

Sample Question:
3. $2 x^{3}+3 x-x^{3}+4 x+5=$ ?
4. $(4 x+3)+\left(x^{2}-4\right)+\left(x^{2}+x\right)=$ ?
5. $\left(3 x^{2}+4 x\right)-(3 x+2)=$ ?
6. $\left(\frac{1}{2}\right)^{2}+\left(\frac{1}{3}\right)^{2}+\left(\frac{1}{4}\right)^{2}=$ ?

## MULTIPLY EXPONENTS

To multiply powers with the same base, add the exponents: $x^{3} * x^{4}=x^{3+4}=x^{7}$
Example: For all $x$, the product $3 x^{2 *} 5 x^{3}=$ ?

Solution: Multiply the coefficients and add the exponents
$3 x^{2} * 5 x^{3}=3 * 5\left(x^{2+3}\right)=15 x^{5}$

Sample Questions:
7. $5 x^{3} y^{5} * 6 y^{2} * 2 x y$ is equivalent to ?
8. The expression $-8 x^{3}\left(7 x^{6}-3 x^{5}\right)$ is equivalent to ?
9. $\left(2 x^{4} y\right)\left(3 x^{5} y^{8}\right)$ is equivalent to ?
10. Which of the following is equivalent to $(x)(x)(x)\left(x^{3}\right)$, for all $x$ ?
a. $4 x$
b. $6 x$
c. $x^{6}$
d. $4 x^{6}$
e. $\quad 4 x^{4}$
11. $4 a^{3} \times 5 a^{8}=$ ?
12. $2 x^{2} * 3 x^{2} y^{2} * 5 x^{2} y$ is equivalent to ?
13. The expression $5 y^{4} * 6 y^{2}$ is equal to which of the following?
a. $\quad 11 y^{2}$
b. $\quad 11 y^{6}$
c. $\quad 11 y^{8}$
d. $\quad 30 y^{6}$
e. $30 y^{8}$

## DIVIDE EXPONENTS

To divide powers with the same base, subtract the exponents: $y^{13} \div y^{8}=y^{13-8}=y^{5}$
Example: For $\mathrm{z} \neq 0$, the expression $\frac{12 z^{10}}{4 z^{2}}$ is equivalent to ?

Solution: Simply the expression by reducing the integers and following the rules for exponents. $12 / 4=3$.
When dividing like bases, subtract the exponents, $\frac{z^{10}}{z^{2}}=z^{10-2}=z^{2}$. The answer is $3 z^{8}$.
Note: The only reason we write $z \neq 0$ is to be proper. $z$ cannot be equal to 0 because any number that is divided by 0 is undefined.

Sample Questions:
14. For all $\mathrm{x}>0$, the expression $\frac{3 x^{3}}{3 x^{9}}$ equals:
15. Reduce $\frac{x^{8} y^{12}}{x^{4} y^{3} z^{2}}$ to its simplest terms.
16. For all a>1, the expression $\frac{3 a^{4}}{3 a^{6}}$ equals:

## RAISE EXPONENTS TO ANOTHER POWER

To raise a power to another power, multiply the exponents: $\left(x^{3}\right)^{4}=x^{3 * 4}=x^{12}$
Example: Which of the following is equivalent to $\left(y^{3}\right)^{8}$ ?
a. $\quad y^{11}$
b. $\quad y^{24}$
c. $\quad 8 y^{3}$
d. $\quad 8 y^{11}$
e. $\quad 24 y$

Solution: When raising a number with an exponent to another power, you multiply the exponents; therefore, $\left(8 y^{3}\right)^{8}=8 y^{3 x 8}=8 y^{24}$ or $C$

Sample Questions:
17. $\left(3 x^{3}\right)^{3}$ is equivalent to ?
18. Which of the following is equivalent to $\left(4 x^{2}\right)^{3}$ ?
a. $64 x^{8}$
b. $64 x^{6}$
c. $12 x^{6}$
b. $12 x^{5}$
c. $4 x^{6}$
19. $\left(n^{5}\right)^{12}$ is equivalent to ?

## MATH 1 LEVEL:

ALGEBRA PROBLEMS WITH EXPONENTS
Use algebra and exponent rules to solve math 1 level problems.
Example: What is the value of $x$ when $5^{x}=125$ ?
Solution: (5)(5)(5) $=125$ so $x$ is 3

Example: $2^{\mathrm{x}}=1 / 16$
Solution: To get a fraction, $x$ must be - number. To get 16,2 must be timed by itself 4 times. So $x=-4$.
Sample Questions:
20. $\left(2 b^{4}\right)^{-1}$
21. $\left(x^{2} y^{-1}\right)^{2}$
22. $\left(n^{3}\right)^{3} * 2 n^{-1}$
23. $\frac{2 y^{3} * 3 x y^{3}}{3 x^{2} y^{4}}$
24. If $x y z \neq 0$, which of the following is equivalent to

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\frac{x^{2} y^{3} z^{4}}{\left(x y z^{2}\right)^{2}} ?
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a. $1 / y$
b. $1 / \mathrm{z}$
c. $y$
d. $x / y z$
e. $x y z$
25. What is the value of the expression $2 x^{3}-x^{2}+3 x+5$ for $x=-2$ ?
26. What is the value of the expression $3 x^{2}+x+4$ for $x=4$

Solve for $x$ in the following:
27. $3^{x}=81$
28. $4^{x}=1 / 64$
29. $2 x^{4}=162$
30. For all real integers, which of the following is NEVER an even number?
I. $x^{3}+3$
II. $2 x+3$
III. $2 x^{2}+3$
a. I only
b. II only
c. III only
d. I and II only
e. II and III only
31. For all real integers, which of the following is ALWAYS an even number?
I. $x^{3}+x-5$
II. $2 x^{2}+4$
III. $2 x^{2}+4$
a. I only
b. II only
c. III only
d. I and II only
e. II and III only
32. Which of the following calculations can yield an even integer for any integer a?
a. $\quad 2 a^{2}+3$
b. $\quad 4 a^{3}+1$
c. $\quad 5 a^{2}+2$
d. $\quad 6 a^{4}+6$
e. $\quad a^{6}-3$

## ADDING AND SUBTRACTING ROOTS

You can add or subtract radical expressions only if the part under the radicals is the same. In other words, treat it like a variable. Just like $2 x+3 x$ would equal $5 x$ :
$2 \sqrt{3}+3 \sqrt{3}=5 \sqrt{3}$
In other words, you can only add or subtract the numbers in front of the square root sign the numbers under the sign stay the same.

## MULTIPLYING AND DIVIDING ROOTS

You can distribute the square root sign over multiplication and division. The product of square roots is equal to the square root of the product:
$\sqrt{3} \times \sqrt{5}=\sqrt{3 x 5}=\sqrt{15}$
The quotient of square roots is equal to the square root of the quotient:
$\frac{\sqrt{6}}{\sqrt{3}}=\sqrt{\frac{6}{3}}=\sqrt{2}$

## SIMPLIFYING SQUARE ROOTS

To simplify a square root, factor out the perfect squares under the radical, square root them, and put the result in front of the part left under the square root sign (the non-perfect- square factors):
$\sqrt{12}=\sqrt{4 \times 3}=\sqrt{4} \times \sqrt{3}=2 \sqrt{3}$
To solve for a power of two, take the square root. To solve for a power of three, take the cube root, etc.:
$x^{2}=16$. Take the square root of both sides: $\sqrt{x^{2}}=\sqrt{16}$ becomes $x=4$
Sample Questions:
33. $3 \sqrt{5}+6 \sqrt{5}=$ ?
34. $\frac{8 \sqrt{18}}{4 \sqrt{2}}=$ ?
35. $3 \sqrt{4} * 5 \sqrt{3}=$ ?
36. $\sqrt{18}=$ ?
37. $5 \sqrt{x}-4 \sqrt{x}$
38. $\frac{5 \sqrt{12}}{\sqrt{3}}=$ ?
39. $2 \sqrt{2} * 3 \sqrt{10}=$ ?
40. $\sqrt{72}=$ ?
41. $4 \sqrt{15} * 2 \sqrt{3}=$ ?
42. $\sqrt{675}=$ ?

## Answer Key

1. 64
2. 64
3. $x^{3}+7 x+5$
4. $2 x^{2}+5 x-1$
5. $3 x^{2}+x+2$
6. $61 / 144$
7. $60 x^{4} y^{8}$
8. $-56 x^{9}+24 x^{8}$
9. $6 x^{9} y^{9}$
10. $x^{6}$
11. $20 a^{11}$
12. $30 x^{6} y^{3}$
13. $30 y^{6}$
14. $\mathrm{x}^{-6}$ or $1 / \mathrm{x}^{6}$
15. $\frac{x^{4} y^{9}}{z^{2}}$
16. $\frac{1}{a^{2}}$ or $\mathrm{a}^{-2}$
17. $27 x^{9}$
18. $64 x^{6}$
19. $n^{60}$
20. $\frac{1}{2 b^{4}}$
21. $\frac{x^{4}}{y^{2}}$
22. $2 n^{8}$
23. $\frac{2 y^{2}}{x}$
24. y
25. -21
26. 56
27. 4
28. -3
29. 3
30. E
31. E
32. D
33. $9 \sqrt{5}$
34. 6
35. $30 \sqrt{3}$
36. $3 \sqrt{2}$
37. $\sqrt{x}$
38. 10
39. $12 \sqrt{5}$
40. $6 \sqrt{2}$
41. $24 \sqrt{5}$
42. $15 \sqrt{3}$
