Name:________________________________ Date:_______________________

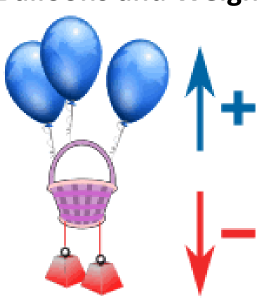
Math 1 Variable Manipulation Part 1
Algebraic Equations

PRE ALGEBRA REVIEW OF INTEGERS (NEGATIVE NUMBERS)
Numbers can be positive (+) or negative (-). If a number has no sign it usually means that it is a positive number. For example: 5 is really +5

Adding Positive Numbers
Adding positive numbers is just simple addition.
Example: 2 + 3 = 5
Solution: This equation is really saying "Positive 2 plus Positive 3 equals Positive 5"
Write it as (+2) + (+3) = (+5)

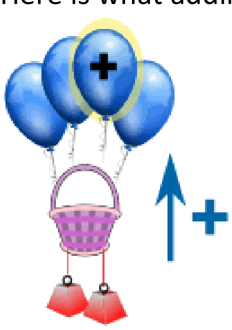
Subtracting Positive Numbers
Subtracting positive numbers is just simple subtraction.
Example: 6 − 3 = 3
Solution: This equation is really saying "Positive 6 minus Positive 3 equals Positive 3"
Write it as (+6) − (+3) = (+3)

Balloons and Weights Visualization

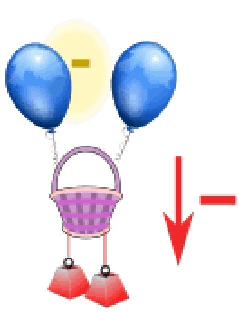


This basket has balloons and weights tied to it:
The balloons pull up (positive)
And the weights drag down (negative)

Here is what adding and subtracting positive numbers looks like:

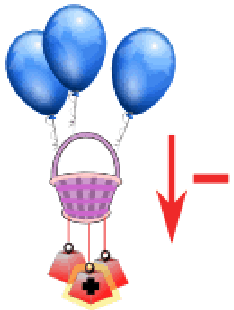


You can add balloons (you are adding positive value)
the basket gets pulled upwards (positive)

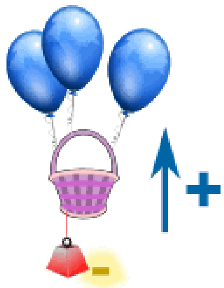


You can take away balloons (you are subtracting positive value)
the basket gets pulled downwards (negative)

Now let's see what adding and subtracting negative numbers looks like:

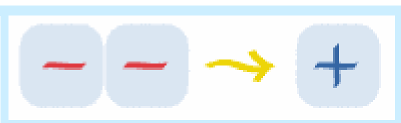


You can add weights (you are adding negative values)
the basket gets pulled downwards (negative)



And you can take away weights (you are subtracting negative values)
the basket gets pulled upwards (positive)

That last one was interesting ... subtracting a negative made the basket go up

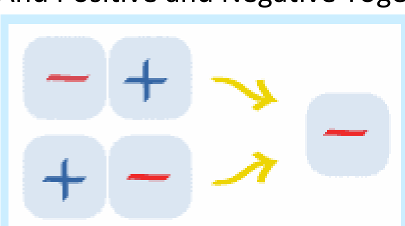


Subtracting a Negative is the same as Adding

Example: What is 6 − (−3) ?
Solution: 6−(−3) = 6 + 3 = 9

Example: What is 14 − (−4) ?
Solution: 14−(−4) = 14 + 4 = 18

We also found that taking away balloons (subtracting positives) or adding weights (adding negatives) both made the basket go down.
And Positive and Negative Together ...



Subtracting a Positive
or
Adding a Negative
is
Subtraction

Example: What is 6 − (+3) ?
Solution: 6−(+3) = 6 **−** 3 = 3

Example: What is 5 + (−7) ?
Solution: 5+(−7) = 5 **−** 7 = −2

The Rules of Integers (negative numbers):
It can all be put into two rules:

	Rule		Example
	Two like signs become a positive sign	+(+)	3+(+2) = 3 + 2 = 5
		−(−)	6−(−3) = 6 + 3 = 9
	Two unlike signs become a negative sign	**+(−)**	7+(−2) = 7 − 2 = 5
		−(+)	8−(+2) = 8 − 2 = 6

They are "like signs" when they are like each other (in other words: the same).

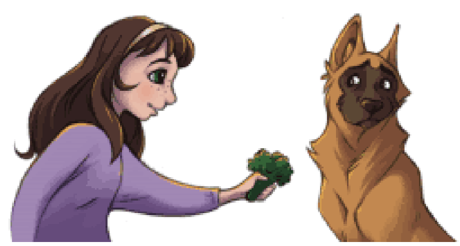
So, all you have to remember is:
Two like signs become a positive sign
Two unlike signs become a negative sign

Example: What is 5+(−2) ?
Solution: +(−) are unlike signs (they are not the same), so they become a negative sign.
5+(−2) = 5 **−** 2 = 3

Example: What is 25−(−4) ?
Solution: −(−) are like signs, so they become a positive sign.
25−(−4) = 25+4 = 29

Example: What is −6+(+3) ?
Solution: +(+) are like signs, so they become a positive sign.
−6+(+3) = −6 **+** 3 = -3

A Common Sense Explanation
And there is a "common sense" explanation:

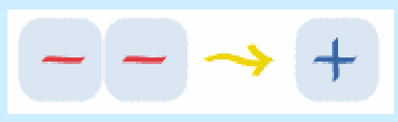


If I say "Eat!" I am encouraging you to eat (positive)
But if I say "Do not eat!" I am saying the opposite (negative).
Now if I say "Do NOT not eat!", so I am back to saying "Eat!" (positive).
So, two negatives make a positive!

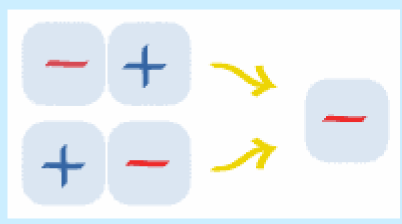
Another Common Sense Explanation
A friend is +, an enemy is −

+ + ⇒ +	a friend of a friend is my friend
+ − ⇒ −	a friend of an enemy is my enemy
− + ⇒ −	an enemy of a friend is my enemy
− − ⇒ +	an enemy of an enemy is my friend

A Bank Example
You have \$80 in your account right now.
Last year the bank subtracted \$10 by mistake, and they want to fix it.
So the bank must take away a negative \$10:
\$80−(−\$10) = \$80 + \$10 = \$90
So you get \$10 more in your account.



So Subtracting a Negative is the same as Adding



So, Subtracting a Positive
or
Adding a Negative
is
Subtraction

Multiplying with integers

You also have to pay attention to the signs when you multiply and divide. There are two simple rules to remember:

1) When you multiply a negative number by a positive number then the product is always negative.
2) When you multiply two negative numbers or two positive numbers then the product is always positive.

This is similar to the rule for adding and subtracting: two minus signs become a plus, while a plus and a minus become a minus. In multiplication and division, however, you calculate the result as if there were no minus signs and then look at the signs to determine whether your result is positive or negative.

Example: $3 \cdot (-4)$
Solution: $3 \cdot (-4) = -12$
3 times 4 equals 12. Since there is one positive and one negative number, the product is negative 12.

Example: $(-3) \cdot (-4)$
Solution: $(-3) \cdot (-4) = 12$
Now we have two negative numbers, so the result is positive.

Dividing Negative Numbers

Rules when dividing by negative numbers:

1) When you divide a negative number by a positive number then the quotient is negative.
2) When you divide a positive number by a negative number then the quotient is also negative.
3) When you divide two negative numbers then the quotient is positive.

Example: $\frac{-12}{3} = ?$

Solution: $\frac{-12}{3} = -4$

Example: $\frac{12}{-3} = ?$

Solution: $\frac{12}{-3} = -4$

Example: $\frac{-12}{-3} = ?$

Solution: $\frac{-12}{-3} = 4$

MATH 1
VOCABULARY FOR ALGEBRAIC EQUATIONS

Variable: Symbols are used to represent unspecified numbers or values. Any letter may be used as a variable.

0.45r 3x + 5 $2 + \frac{Y}{3}$ v * w $\frac{1}{2}pq \div 4rs$

Constant: A term whose value does not change.

5x + 4 (4 is the constant)

Coefficient: The numerical factor of a term.

6x (6 is the coefficient)

Term: May be a number, a variable, or a product or quotient of numbers and variables.

3x – 5 = -7x + 10 (This equation has 4 terms: 3x, -5, -7x, and +10)

Like Terms: Terms that contain the same variables, with corresponding variables having the same power.

$2x^3y^2$ and $-4\,x^3y^2$ are like terms

Algebraic Expression: Consists of sums and/or products of number and variables. A variable or combination of variables, numbers, and symbols that represents a mathematical relationship.

$\frac{3x}{2} + 5 - x^2$

Equation: A mathematical statement that contains an equal sign.

9x – 8 = 22 - x

Solution: A set of elements that make an equation or inequality true. Any and all value(s) of the variable(s) which satisfies an equation, or inequality.

The only solution for the equation 2x – 15 = -3 is x = 4 .

ONE STEP EQUATIONS

To solve an equation means to find the value of x that makes the equation true. In order to maintain equality the equation must be balanced, meaning what is done to one side must be done to the other side. Usually when you solve an equation, you start with the equation and find what x equals.

There are four operations to perform to solve algebraic equations, +, -, ×, or ÷. Use the appropriate operation to get x alone on one side of the equation and everything else on the other side of the equation.

Addition Example: If x + 4 = 8, what is the value of x?
Solution: This equation is easy enough that we can guess the value of x to be 3. However, to solve it algebraically, we do the OPPOSITE of add 4, so we subtract 4 from each side of the equation to get x alone.

x + 4 = 8
  -4 -4
x = 4

Subtraction Example: If x - 4 = 8, what is the value of x?
Solution: This equation is easy enough that we can guess the value of x to be 11. However, to solve it algebraically, we do the OPPOSITE of add 4, so subtract 4 from each side of the equation to get x alone.

x - 4 = 8
  +4 +4
x = 11

Multiplication Example: If 4x = 8, what is the value of x?
Solution: This equation is easy enough that we can guess the value of x to be 11. However, to solve it algebraically, we do the OPPOSITE of multiply by 4, so divide by 4 from each side of the equation to get x alone.

$4x = 8$

$\frac{4x}{4} = \frac{8}{4}$

$x = 2$

Multiplication Example #2: Solve for x in the equation: $\frac{3}{4}x = 12$

Solution: To solve one step multiplication problems, we do the OPPOSITE of multiply by $\frac{3}{4}$, so divide by $\frac{3}{4}$ from each side of the equation to get x alone. Remember that to divide by a fraction, we multiply by the reciprocal.

$\frac{3}{4}x = 12$

$\frac{4}{3} \times \frac{3}{4}x = 12 \times \frac{4}{3}$

$x = 16$

Division Example: If $\frac{x}{4} = 8$, what is the value of x?

Solution: This equation is easy enough that we can guess the value of x to be 11. However, to solve it algebraically, we do the OPPOSITE of divide by 4, so multiply by 4 from each side of the equation to get x alone.

$\frac{x}{4} = 8$

$4 \times \frac{x}{4} = 8 \times 4$

$x = 32$

Sample Questions:

1. $6t = 30$

2. $x - 6 = 4$

3. $\frac{x}{4} = 12$

4. $5 + x = 10$

5. $\frac{2}{5}x = 10$

TWO STEP EQUATIONS

There are two ways to handle 2-step equations. The first way is to use basic operations to get the **term with the x** by itself on one side of the equation. The use the appropriate one step equation to solve for x.

Example: 7 – 5c = 37

Solution: Subtract 8 from both sides to get the term with the c (-5c) by itself. Then divide by -5 to solve for c.

7 - 5c = 37
-7 -7
-5c = 30
$\frac{-5c}{-5} = \frac{30}{-5}$
C = -6

Example: 4x – 3 = 17

Solution: First add three to each side leaving 4x one on side of the equation. Then divide both sides by 4.

4x – 3 = 17
+3 +3
4x = 20
$\frac{4x}{4} = \frac{20}{4}$
x = 5

Example: $6 = \frac{a}{4} + 2$

Solution: First subtract 2 from both sides of the equation to get the a/4 term by itself. Then multiply both sides of the equation by 4 to get a alone.

$6 = \frac{a}{4} + 2$
-2 -2
$4 = \frac{a}{4}$
$4 \times 4 = \frac{a}{4} \times 4$
16 = a

The second way is used if the term with the variable **and** another term are both multiplied or divided by a number, then divide or multiply to get the variable and connected term alone and then add or subtract to get the variable alone.

Example: 2(n + 5) = -2

Solution: First divide both sides of the equation by 2, then subtract 5 from both sides of the equation.

2(n + 5) = -2

Example: $\frac{x+1}{3} = 2$

Solution: First multiply both sides of the equation by 3 and then subtract 1 from each side.

$\frac{x+1}{3} = 2$
$3 \times \frac{x+1}{3} = 2 \times 3$
x+1 = 6
-1 -1
x = 5

Sample Questions:

6. -15 = -4m + 5
7. 10 – 6v = -104
8. 8n + 7 = 31
9. -9x -13 = -103
10. $\frac{n+5}{-16}$ = -1
11. -10 = -10 + 7m
12. -10 = 10(k – 9)
13. $\frac{m}{9}$ - 1 = -2
14. 9 + 9m = 9
15. 7(9 + k) = 84
16. 8 + $\frac{b}{-4}$ = 5

DISTRIBUTIVE PROPERTY

The distributive property is one of the most frequently used properties in math. It lets you multiply a sum by multiplying each addend separately and then add the products. It is easier to understand the meaning by looking at the examples below.

$$5\,(x + 2) = 5 \bullet x \; + \; 5 \bullet 2$$

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$$(5x)(3x + 6) = 5x \bullet 3x \; + \; 5x \bullet 6$$

$$(5)(3x^2 + 2x + 6) = 5 \bullet 3x^2 + 5 \bullet 2x + 5 \bullet 6$$

Consider the first example, the distributive property lets you "distribute" the 5 to both the 'x' and the '2'.

Example: 5(n + 6)
Solution: Multiply 5 by both n and 6
5(n + 6)
5*n +5*6
5n + 30

Example: -7(5k – 4)
Solution: Multiply both 5k and -4 by -7
-7(5k – 4)
-7*5k – (-7)(4)
-35k + 28

Sample Questions:

17. 4(3r – 8)

18. (5 + 5x)*3

19. -10(1 + 9x)

20. -5(7a – 6)

ADDING AND SUBTRACTING MONOMIALS {COMBINING LIKE TERMS)

Combining like terms is just a way to count the number of items that are the same. Another way is to keep the variable part unchanged while adding or subtracting the coefficients (numbers):

Example: 2a + 3a
Solution: There are 2 a's and another 3 a's, so 2 + 3 = 5 so there are a total of 5 a's

Example: 2a + 3a
Solution: Add the coefficients and keep the variables the same.
2a + 3a = (2 + 3)a = 5a

Example: For all x, 13 – 2(x + 5) = ?
Solution: First distribute the -2 and then combine like terms
13 – 2(x + 5)
13 - 2x - 10
13 – 10 – 2x
3 – 2x

Sample Question:

21. -18 – 6k + 6(1 + 3k)

22. 5n + 34 -2(1 – 7n)

23. 2(4x – 3) – 8 + 4 – 2x

24. 3n – 5 -8(6 + 5n)

25. -3(4x + 3) +4(6x + 1) - 43

MULTI-STEP EQUATIONS

The steps to solving a multi-step equation are:

1. Distribute
2. Collect like terms
3. Gather all the variables on one side
4. Gather all the constants on other side
5. The variable should have a coefficient of 1
6. Check your answer

Example: 5p – 14 = 8p + 4

Solution: Subtract 5p from both sides. Subtract 4 from both sides. Then divide both sides by 3.

5p – 14 = 8p + 4
-5p   -5p
-14 = 3p + 4
-4   -4
$\frac{-18}{3} = \frac{3p}{3}$
-6 = p

Example: p – 1 = 5p + 3p - 8

Solution: Add 5p and 3p to get 8p. Subtract p from both sides and add 8 to both sides. Then divide both sides by 7.

p – 1 = 5p + 3p – 8
p – 1 = 8p – 8
-p   -p
-1 = 7p – 8
+8   +8
$\frac{7}{7} = \frac{7p}{7}$
1 = p

Example: 12 = -4(-6x – 3)

Solution: Distribute -4 into the parentheses. Subtract 12 from each side of the equation. Divide both sides of the equations by 24.

12 = -4(-6 – 3)
12 = 24x + 12
-12   -12
$\frac{0}{24} = \frac{24x}{24}$
0 = x

Zero can be the answer to an equation (as seen in the example above). It is also possible to have "No Solution". This happens when both sides can NEVER be equal.

Example: 4m – 4 = 4m

Solution: Subtract 4m from both sides and you get -4 = 0 which is not true. -4 ≠ 0 so there is No Solution.

A negative outside the parentheses means to distribute the negative, or a negative 1, into the parentheses. If the variable is by itself, but is negative, divide both sides by negative 1 to get a positive variable.

Example: 14 = -(p – 8)
Solution: Distribute the negative (or negative 1) into the parentheses. Subtract 8 from both sides. Divide both sides by -1.
14 = -(p – 8)
14 = -p + 8
-8 -8
6 = -p
-1 -1
-6 = p

LIST OF ALGEBRAIC PROPERTIES

Property (a, b and c are real numbers)	**Example**
Identity Property of Addition Any number when added to 0 is itself.	$c+0=c$
Identity Property of Multiplication Any number multiplied by 1 is itself.	$b\cdot 1=b$
Multiplicative Property of Zero Any number multiplied by zero is zero.	$a\cdot 0=0$
Commutative Property of Addition or Multiplication The order in which you add or multiply does not change the sum or product.	$a+b=b+a$ $ab=ba$
Associative Property of Addition or Multiplication The order in which you group numbers does not change the sum or product.	$(a+b)+c=a+(b+c)$ $(ab)c=a(bc)$
Symmetric Property If one number is equal to a second number, then the second number is equal to the first.	If $a=b$ then $b=a$
Substitution Property If $a = b$ then a can be substituted into an equation for b.	If $a=b$ and $b+c=6$ then $a+c=6$.
Distributive Property The product of a number and a sum or difference is the same as the sum or difference of the products of the number and each element of the sum or difference.	$a(b+c)=ab+ac$ $a(b-c)=ab-ac$
Additive Inverse The sum of a number and its inverse is zero.	$a+(-a)=0$
Multiplicative Inverse The product of a number and its inverse is one.	$b\cdot\frac{1}{b}=1$
Addition Property of Equality Adding the same number to each side of an equation produces an equivalent equation.	If $a=b$ then $a+c=b+c$
Subtraction Property of Equality Subtracting the same number from each side of an equation produces an equivalent equation.	If $a=b$ then $a-c=b-c$
Multiplication Property of Equality Multiplying each side of the equation by the same number produces an equivalent equation.	If $a=b$ then $ac=bc$
Division Property of Equality Dividing each side of the equation by the same number produces an equivalent equation.	If $a=b$ then $\frac{a}{c}=\frac{b}{c}$

Sample Questions:

26. $-8 = -(x + 4)$

27. $-18 – 6k = 6(1 + 3k)$

28. $5n + 34 = -2(1 – 7n)$

29. $2(4x – 3) – 8 = 4 + 2x$

30. $3n – 5 = -8(6 + 5n)$

31. $–(1 + 7x) -6(-7 –x) = 36$

Answer Key

1. 5
2. 10
3. 48
4. 5
5. 25
6. 5
7. 19
8. 3
9. 10
10. 11
11. 0
12. 8
13. -9
14. 0
15. 3
16. 12
17. 12r – 32
18. 15 + 15x
19. -10 – 90x
20. -35a + 30
21. -12 – 3k or 12k - 12
22. 19n + 32
23. 6x – 10
24. 53 – 37n or -37n - 53
25. 12x -48
26. 4
27. -1
28. 4
29. 3
30. -1
31. 5